South Australian
Certificate of Education

General Mathematics
2018

Question booklet

- Questions 1 to 9
- Answer ***all*** questions
- Write your answers in this question booklet
- You may write on page 21 if you need more space

Examination information

Materials

- Question booklet
- SACE registration number label

Reading time

- 10 minutes
- You may begin writing during this time
- You may begin using an approved calculator during this time

Writing time

- 2 hours
- Show appropriate working and steps of logic in this question booklet
- Use black or blue pen
- You may use a sharp dark pencil for diagrams
- Approved calculators may be used — complete the box below

Total marks 90

Government of South Australia

Attach your SACE registration number label here	**Graphics calculator** 1. Brand _____ Model _____ 2. Brand _____ Model _____

Question 1 (8 marks)

Dimitrios needs $\$150\,000$ to set up a new business. He obtains an interest-only loan from a financial institution that charges a flat interest rate of 4.3% per annum, payable half yearly. Dimitrios is setting up a sinking fund that he can use to repay the $\$150\,000$ in full in 3 years' time.

(a) Calculate the half-yearly interest payment for the interest-only loan.

(1 mark)

(b) The sinking fund earns interest of 3.7% per annum, compounded monthly.

Show that the amount that Dimitrios must pay into the sinking fund each month in order to save the $\$150\,000$ in 3 years is approximately $\$3940$.

(2 marks)

(c) Calculate the total cost of the interest-only loan and sinking fund option.

(2 marks)

(d) As an alternative option, Dimitrios could have borrowed the $\$150\,000$ using a reducing-balance (principal and interest) loan with an interest rate of 3.9% per annum, compounded quarterly.

Calculate the total cost of this option, and compare it with the interest-only loan and sinking fund option in part (c).

(3 marks)

Question 2 (10 marks)

A population of blacklip abalone (large edible sea snails) exists on a reef off the coast of Tasmania. A random sample of abalone from this population was collected, and their lengths were measured. The distribution of lengths was found to be normal, with a mean of 110 millimetres and a standard deviation of 20 millimetres.

(a) The minimum legal length for possession of a blacklip abalone is 130 millimetres.

State the proportion of blacklip abalone that could be expected to be shorter than this.

(1 mark)

(b) Marine scientists believe that only the longest 60% of blacklip abalone are capable of reproduction.

Show that the shortest blacklip abalone capable of reproduction is approximately 105 millimetres.

(2 marks)

(c) The total population of blacklip abalone on the reef is estimated to be 3500.

How many of these abalone could be expected to be long enough to reproduce but shorter than the minimum legal length?

(2 marks)

On the same reef is another species of abalone, known locally as Thompson's abalone. The lengths of Thompson's abalone are normally distributed, with a mean of 140 millimetres and a standard deviation of 30 millimetres. It is estimated that the population size is approximately the same as that of the blacklip abalone.

The minimum legal length for possession of a Thompson's abalone is 165 millimetres.

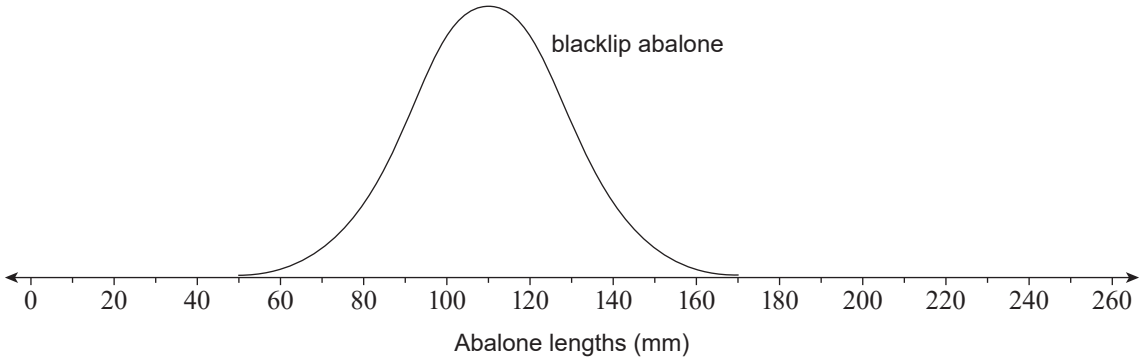
(d) If an abalone of legal length is collected from the reef, which species (blacklip or Thompson's) is it more likely to be? Justify your answer.

(2 marks)

(e) The graph below shows the probability distribution of the lengths of blacklip abalone.

On the scaled axis below, add a graph to represent the probability distribution of the lengths of Thompson's abalone.

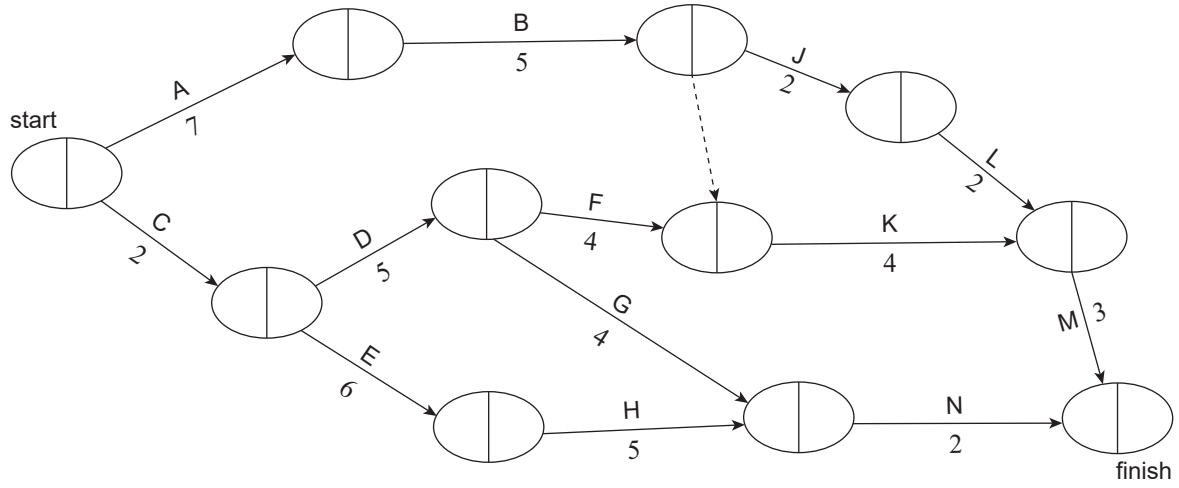
Lengths of abalone



(3 marks)

Question 3 (13 marks)

Adam wants to install a festive lights display on his house, and he has identified the tasks that need to be completed. The times needed (in days) to complete each task, and the order for completion, are shown in the following network diagram:



(a) List all the tasks that must be completed before task K can begin.

(2 marks)

(b) *Using the network diagram above*, complete a forward and backward scan.

(2 marks)

(c) (i) List all the tasks that are on the critical path(s).

(2 marks)

(ii) Why are these tasks considered critical?

(1 mark)

(iii) If Adam wants to finish the installation of his festive lights display on 30 November, what is the latest date on which he can start the installation?

(1 mark)

(iv) State *one* task that could be delayed by 2 days and not result in an increase in the minimum completion time. Use figures from the network diagram on page 6 to justify your answer.

(2 marks)

(v) Adam's friend Trang has time to help with only *one* of the tasks, and she has offered to help with task L, task M, or task N. Trang's help would reduce the time for any of these tasks by 1 day.

For each task (L, M, and N), state whether or not Adam should choose to have Trang help. Justify each answer. *No further forward and backward scans of the network are required.*

(1) Task L

(1 mark)

(2) Task M

(1 mark)

(3) Task N

(1 mark)

Question 4 (10 marks)

Eco Homes, a house-construction business, offers 10 different house designs. The floor area (in square metres) is found to correlate with the price (in Australian dollars) of building the house.

Floor area (A m^2)	176	186	120	126	226	248	264	150	165	264
Price (P AU$)	203 200	205 000	173 300	176 200	228 100	234 000	235 200	185 400	193 100	238 700

The linear regression equation relating the two variables (floor area and price) is $P = 457.91A + 119\,072$.

(a) Interpret the meaning of the value '457.91' in this equation in the context of the question.

(2 marks)

(b) State the value of Pearson's correlation coefficient (r), and interpret its meaning.

(3 marks)

(c) Volaris wants to have a house built, and he has $\$210\,000$ to spend. Eco Homes has agreed to design and build a house for him.

(i) Using the linear regression model, determine the floor area of the largest Eco Homes house that Volaris could expect to purchase for $\$210\,000$.

(1 mark)

(ii) Volaris now decides that he needs a house with a floor area of 135 m^2, and he contracts Eco Homes to design and build it for him. Approximately how much can Volaris expect it to cost?

(1 mark)

(d) Eco Homes is adding a new house design to its range. The new house design will have a floor area of 332 m^2 and will cost $\$322\,040$.

How will adding the data for the new house design to the existing data affect the following parameters of the linear regression model? For each parameter, tick the appropriate box to indicate your answer.

(i) The slope will:

increase ☐ stay the same ☐ decrease ☐ (1 mark)

(ii) The y-intercept will:

increase ☐ stay the same ☐ decrease ☐ (1 mark)

(iii) The value of Pearson's correlation coefficient (r) will:

increase ☐ stay the same ☐ decrease ☐ (1 mark)

Question 5 (13 marks)

Laura is a 23-year-old who is setting up her first superannuation account. Laura's employer contributes $\$1235$ to her superannuation fund each quarter. Laura's superannuation fund pays 6.8% interest per annum, compounded quarterly.

(a) (i) Show that when Laura turns 45 she will have approximately $\$250\,000$ in her superannuation account.

(2 marks)

(ii) Calculate the interest that will have been earned in Laura's superannuation account when Laura turns 45.

(1 mark)

(b) (i) If Laura retires when her superannuation account balance reaches $\$1\,500\,000$, how old would she be when she retires?

(2 marks)

(ii) State *one* assumption (apart from the interest rate remaining constant) that is being made when doing the calculation in part (b)(i), and explain how it affects the answer.

(2 marks)

When Laura retires, she decides to invest the $\$1\,500\,000$ balance of her superannuation account in an annuity. She has identified two options:

- Option A pays 3.75% interest per annum, compounded weekly.
- Option B pays 3.76% interest per annum, compounded monthly.

(c) (i) (1) Calculate the effective interest rate for Option A.

(1 mark)

(2) Calculate the effective interest rate for Option B.

(1 mark)

(ii) State *one* reason why Laura might choose to invest in Option A.

(1 mark)

(iii) State *one* reason why Laura might choose to invest in Option B.

(1 mark)

(d) Laura decides not to invest in Option A or Option B. Instead she invests the $\$1\,500\,000$ in Option C, which pays 3.78% interest per annum, compounded fortnightly.

Calculate how much Laura will be paid fortnightly in retirement, if she wants her annuity to last 25 years, and wants to have $\$100\,000$ remaining in the account at the end of that time.

(2 marks)

Question 6 (11 marks)

A shopping centre has four children's rides (car, ship, tractor, and horse) that are located outside four shops: Shop A, Shop B, Shop C, and Shop D. Centre management has previously rotated the rides among the four shops, and the average weekly profits (in dollars) for each ride at each shop are shown in the table below.

Ride	*Shop A*	*Shop B*	*Shop C*	*Shop D*
Car	30	38	30	32
Ship	20	38	33	24
Tractor	50	45	50	58
Horse	32	40	30	35

(a) Which children's ride appears to be the most popular?

(1 mark)

Centre management wants to assign the rides to the shops to make the maximum total profit per week from the rides.

(b) Complete the array below for the *first step* of the Hungarian algorithm, used to find the *maximum* total profit per week.

28	20	28	26
38	20	25	34
8	13		
26	18		

(1 mark)

(c) (i) From the reduced array below, state the optimal assignment(s) of rides to shops to make the maximum total profit per week.

0	0	3	6
10	0	0	14
0	13	3	0
0	0	5	5

(2 marks)

(ii) State the maximum total profit that can be made per week.

(1 mark)

Question 6 continues on page 14.

The tractor ride has broken down and will not be replaced. Hence the array becomes:

Ride	*Shop A*	*Shop B*	*Shop C*	*Shop D*
Car	30	38	30	32
Ship	20	38	33	24
Horse	32	40	30	35

(d) (i) Use the Hungarian algorithm to determine which shop will no longer have a ride placed outside it, if the maximum total profit per week is to be made. Show your working in the space below.

(5 marks)

(ii) What is the maximum total profit that can now be made per week from the three remaining rides?

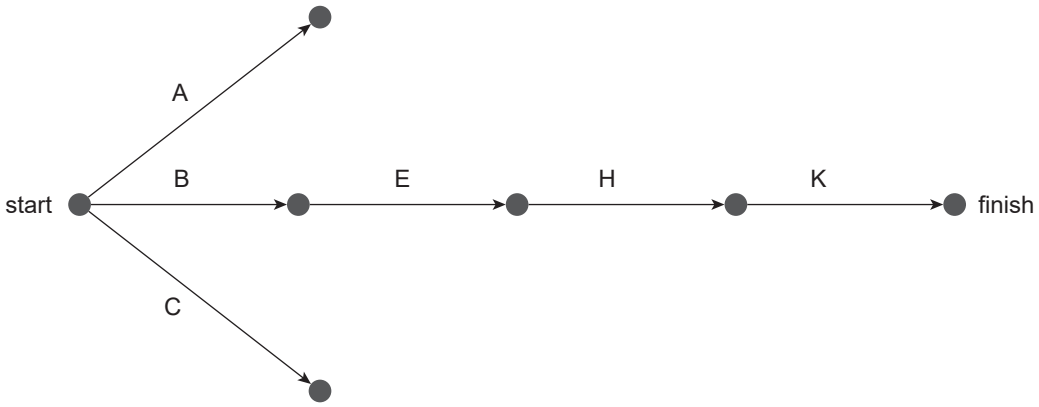
(1 mark)

Question 7 (5 marks)

Nirmala is planning to renovate her shop. The precedence table below shows the tasks, A to K, required to complete the renovations, the times to complete each task, and the prerequisites for each task.

Task	*Time (days)*	*Prerequisites*
A	3	–
B	3	–
C	4	–
D	2	A, B
E	5	B
F	4	C
G	4	D
H	3	E, F
I	5	G, H, J
J	2	C
K	4	G, H, J

(a) Using the information in the precedence table above, complete the network diagram below. Indicate any dummy link(s) with a dotted line.



(3 marks)

(b) Nirmala's goal is to complete the renovation within 14 days.

State whether this is possible. Justify your answer. *A forward and backward scan is **not** required.*

(2 marks)

Question 8 (10 marks)

A couple has borrowed $\$320\,000$ from a bank to buy their first house. The term of the loan is 30 years, and the interest rate for the loan is 4.5% per annum, compounded weekly.

(a) Calculate the couple's weekly repayment.

(2 marks)

(b) Show that the couple's outstanding debt after the first 5 years will be approximately $\$291\,700$.

(1 mark)

After the first 5 years, the couple inherits $\$10\,000$. They deposit this money in an offset account that is attached to their loan account.

(c) (i) Show that it will take approximately 1220 weeks for the couple to pay off the outstanding debt, if they keep the $\$10\,000$ in the offset account until the end of the loan.

(1 mark)

(ii) Find the approximate amount of interest that the couple will save by using the offset account.

(2 marks)

(d) The couple could have used their inheritance to invest in a term deposit or to pay a lump sum off the principal owing on the loan.

State *one* advantage of using an offset account instead of one of these options.

(1 mark)

(e) (i) Calculate the couple's outstanding debt after they have had the offset account for 6 years.

(1 mark)

(ii) The couple decides to close the offset account and spend the money on a holiday.

Calculate the couple's new outstanding debt.

(1 mark)

(iii) Calculate how long it will now take the couple to pay off the outstanding debt.

(1 mark)

Question 9 (10 marks)

The values (in US$) and the weights (in carats) for 10 high-quality diamonds are shown below.

Weight (carats)	0.19	0.20	0.21	0.30	0.31	0.52	0.53	0.71	0.75	1.01
Value (US$)	677	616	643	911	1149	2443	2745	4312	5158	11 206

Source: Adapted from Hamblyn, C 2018, 'Diamonds are forever' in 'Data sets for bivariate investigations', CensusAtSchool NZ, viewed 12 April 2018, http://new.censusatschool.org.nz/resource/multivariate-data-sets/

(a) Determine the linear regression equation for the data, in the form $y = ax + b$.

(1 mark)

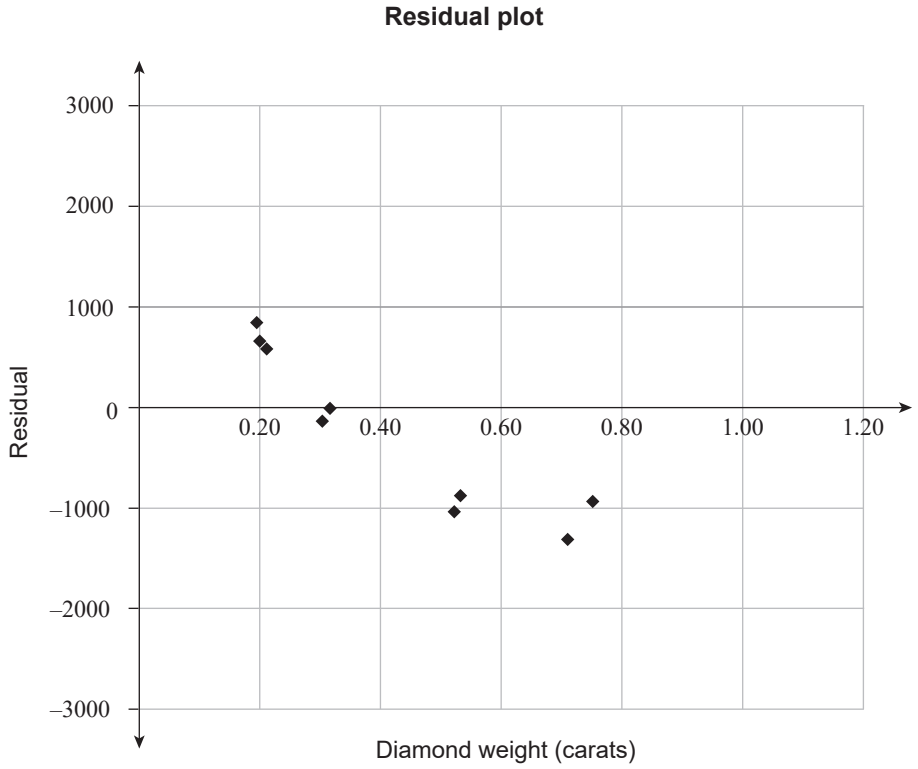
(b) Using the linear regression equation that you determined in part (a), predict the value (in US$) of a diamond with a weight of 0.21 carats.

(1 mark)

(c) Discuss the reliability of the linear regression model for predicting from this data. Give at least *two* reasons for your answer.

(2 marks)

A residual plot for the linear regression was calculated and is given below:



(d) One of the points is missing from the residual plot.

(i) State the coordinates of the missing point in the space below.

(1 mark)

(ii) Plot the missing point on the residual plot above. (1 mark)

Question 9 continues on page 20.

It is decided that an exponential equation is a more suitable model for the data.

(e) Find the exponential regression model for the data, in the form $y = a \times b^x$.

(1 mark)

(f) (i) Using the exponential model, calculate the predicted weight of a diamond valued at $\$7500$. Give your answer correct to two decimal places.

(1 mark)

(ii) Comment on the reliability of the prediction made in part (f)(i), and use mathematical reasoning to justify your answer.

(2 marks)

You may write on this page if you need more space to finish your answers. Make sure to label each answer carefully (e.g. 6(c)(i) continued).