STAGE 2 GENERAL MATHMATIC
ASSESSMENT TYPE 2: MATHEMATICAL INVESTIGATION
THE HUNGARIAN ALGORITHM

(Note: When reading investigation read Right -> Left -> Down)

An interschool swimming carnival came up in the agender for the vested city private school. The school has asked the swimming coach to see if there is was any volunteer among the students to swim in the 4 x 50 meters medley relay. There were nine students who decide to put their names down for the event, Asher, Brandon, Cameron, Dylan, Ethan, Finn, George, Harry and Isaac. Sadly, only four students could make the team. The coach had to decide how to form the best and fastest team and whether to create a second team for the race.

The coach asked the school to have a trial swimming session for the volunteers, so he could time each student to see how fast they swam in each stroke. The data recorded on the day for each of the nine students in each stroke of the relay is in the graph below. The data was recorded in whole second to ensure that the coach could have an easier time picking each time.

	Asher	Brandon	Cameron	Dylan	Ethan	Finn	George	Harry	Isaac
Freestyle	34	35	38	28	32	38	35	33	35
Breaststroke	41	44	45	45	46	46	47	43	46
Backstroke	41	42	37	42	42	43	40	35	41
Butterfly	34	38	38	36	36	41	40	37	41

After looking puzzled for some time and not knowing which students to select for the race, the maths teacher gives a suggestion to use the Hungarian algorithm to solve which student would be best in each stroke of the race. The coach decides to put the first four student who put their names down first, Asher, Brandon, Cameron and Dylan in the relay team. The coach decides to call the team (T1)

Method for the Hungarian Algorithm

For the method for the algorithm we can use a set an example for easier explanation, consider the example is where four jobs (J1, J2, J3 and J4) need to be taken up by four works (W1, W2, W3 and W4), only one worker can have one of the four jobs. Each worker was giving four tests for the four jobs, the tests came back with each worker having a test score of (1-100) for each job. A matrix is used to display the tests score and is used to work out the Hungarian Algorithm.

$$\text{Matrix example} = \begin{bmatrix} & J1 & J2 & J3 & J4 \\ W1 & 82 & 83 & 69 & 92 \\ W2 & 77 & 37 & 49 & 92 \\ W3 & 11 & 69 & 5 & 86 \\ W4 & 8 & 9 & 98 & 23 \end{bmatrix}$$

Step 1: Subtracting the rows minimum

Start by subtracting the row minimum from each of the rows. The smallest number in the first row is, for example, 69. Therefore, 69 is subtracted from each of the other numbers in the first row. The same rule applies for all the four rows.

$$\begin{bmatrix} & J1 & J2 & J3 & J4 \\ W1 & 82 & 83 & 69 & 92 \\ W2 & 77 & 37 & 49 & 92 \\ W3 & 11 & 69 & 5 & 86 \\ W4 & 8 & 9 & 98 & 23 \end{bmatrix} \begin{matrix} (-69) \\ (-37) \\ (-5) \\ (-8) \end{matrix} \text{ Result} = \begin{bmatrix} & J1 & J2 & J3 & J4 \\ W1 & 13 & 14 & 0 & 23 \\ W2 & 40 & 0 & 12 & 55 \\ W3 & 6 & 64 & 0 & 81 \\ W4 & 0 & 1 & 90 & 15 \end{bmatrix}$$

Step 2: Subtracting the columns minimum

This step is like the last step but instead of subtracting the smallest number in each row, the smallest number in each column is subtracted from each column. For example, 0 is the smallest number in the first column. Therefore, 0 is subtracted from each of the other numbers in the first column. The same rule applies for all the four columns.

$$\begin{bmatrix} & J1 & J2 & J3 & J3 \\ W1 & 13 & 14 & 0 & 23 \\ W2 & 40 & 0 & 12 & 55 \\ W3 & 6 & 64 & 0 & 81 \\ W4 & 0 & 1 & 90 & 15 \\ & (-0) & (-0) & (-0) & (-15) \end{bmatrix} \text{Result} = \begin{bmatrix} & J1 & J2 & J3 & J4 \\ W1 & 13 & 14 & 0 & 8 \\ W2 & 40 & 0 & 12 & 40 \\ W3 & 6 & 64 & 0 & 66 \\ W4 & 0 & 1 & 90 & 0 \end{bmatrix}$$

Step 3: Covering all the zeros with a minimum number of lines

This step determines the minimum number of lines (horizontal and vertical) that are needed to cover all the zeros in the matrix. All the zeros can be covered in 3 lines.

$$\begin{bmatrix} & J1 & J2 & J3 & J4 \\ W1 & 13 & 14 & 0 & 8 \\ W2 & 40 & 0 & 12 & 40 \\ W3 & 6 & 64 & 0 & 66 \\ W4 & 0 & 1 & 90 & 0 \end{bmatrix}$$

Because the number of lines needed (3) is lower than the size of the matrix, which is a 4x4, step 4 is needed to make sure that there are 4 line, so the number of lines is the same as the matrix.

Step 4: Create additional zeros

First, the smallest number which is not covered by a line is 6. 6 is subtracted from all the other uncovered number, then 6 is added to all the number which are covered twice by a line (this mean a two line are covering a single number) Numbers that are covered by one line, no subtraction or addition is needed.

$$\begin{bmatrix} & J1 & J2 & J3 & J4 \\ W1 & 7 & 8 & 0 & 2 \\ W2 & 40 & 0 & 18 & 40 \\ W3 & 0 & 58 & 0 & 60 \\ W4 & 0 & 1 & 96 & 0 \end{bmatrix}$$

Step 3: Covering all the zeros with a minimum number of lines

Again, determining the minimum number of lines that are needed to cover all the zeros in the matrix. All the zeros can be covered in 4 lines.

$$\begin{bmatrix} & J1 & J2 & J3 & J4 \\ W1 & 7 & 8 & 0 & 2 \\ W2 & 40 & 0 & 18 & 40 \\ W3 & 0 & 58 & 0 & 60 \\ W4 & 0 & 1 & 96 & 0 \end{bmatrix}$$

Because the number of lines needed is 4 and is equal to the size of the matrix, an assignment can be found using the zeros within the matrix. Therefore, the algorithm stops.

One of the assignments could be $\begin{bmatrix} & J1 & J2 & J3 & J4 \\ W1 & 7 & 8 & \boxed{0} & 2 \\ W2 & 40 & \boxed{0} & 18 & 40 \\ W3 & \boxed{0} & 58 & 0 & 60 \\ W4 & 0 & 1 & 96 & \boxed{0} \end{bmatrix}$

(Note there can be one or more assignment for each matrix)

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The coach decided to predict which students would be selected for each stroke. The prediction is based on which of the students are the fastest in each stroke of the relay. The team that is predicted to be selected is:

Freestyle = Cameron
Breaststroke = Brandon
Backstroke = Asher
Butterfly = Dylan

For each student the current times that they hold for each stroke is:

Freestyle = Cameron = 38 seconds
Breaststroke = Brandon = 44 seconds
Backstroke = Asher = 41 seconds
Butterfly = Dylan = 36 seconds

The total time for all student to complete the relay is:
$38 + 44 + 41 + 36 = 159$
The total time is 159 seconds which is 2 minutes and 39 seconds.

The coach decides to determine which of these students should swim each stoke using the Hungarian Algorithm.

(*Original Matrix*)
This is the original matrix:

Matrix T1 = $\begin{bmatrix} 34 & 35 & 38 & 28 \\ 41 & 44 & 45 & 45 \\ 41 & 42 & 37 & 42 \\ 34 & 38 & 38 & 36 \end{bmatrix}$

(*Subtracting rows minimum*)
We subtract the row minimum from each row:

$\begin{matrix} 34 & 35 & 38 & 28 & (-28) \\ 41 & 44 & 45 & 45 & (-41) \\ 41 & 42 & 37 & 42 & (-37) \\ 34 & 38 & 38 & 36 & (-34) \end{matrix}$ $Result$ $\begin{matrix} 6 & 7 & 10 & 0 \\ 0 & 3 & 4 & 4 \\ 4 & 5 & 0 & 5 \\ 0 & 4 & 4 & 2 \end{matrix}$

(*Subtracting columns minimum*)
Because each column contains a zero, subtracting minimum is not needed:

$\begin{matrix} 6 & 7 & 10 & 0 \\ 0 & 3 & 4 & 4 \\ 4 & 5 & 0 & 5 \\ 0 & 4 & 4 & 2 \\ (-0) & (-0) & (-0) & (-0) \end{matrix}$ $Result$ $\begin{matrix} 6 & 4 & 10 & 0 \\ 0 & 0 & 4 & 4 \\ 4 & 2 & 0 & 5 \\ 0 & 1 & 4 & 2 \end{matrix}$

(*Covering all the zeros with a minimum number of lines*)
4 line are needed to cover all the zeros:

$\begin{matrix} 6 & 4 & 10 & 0 \\ 0 & 0 & 4 & 4 \\ 4 & 2 & 0 & 5 \\ 0 & 1 & 4 & 2 \end{matrix}$

(*The assignment*)
Because there are 4 lines needed, the zeros show an assignment:

Matrix T1 = $\begin{matrix} 6 & 4 & 10 & \mathbf{0} \\ 0 & \mathbf{0} & 4 & 4 \\ 4 & 2 & \mathbf{0} & 5 \\ \mathbf{0} & 1 & 4 & 2 \end{matrix}$

(*T1 Results*)
Freestyle: Dylan
Breaststroke: Brandon
Backstroke: Cameron
Butterfly: Asher
∴
Freestyle: 28
Breaststroke: 44
Backstroke: 37
Butterfly: 34

The total time for all student to complete the relay is:
$28 + 44 + 37 + 34 = 143$

The total time is 143 seconds which is 2 minutes and 23 seconds.

The prediction was based on which student was the fastest in each style/stroke.
Prediction Results:

Freestyle = Cameron
Breaststroke = Brandon
Backstroke = Asher
Butterfly = Dylan
∴
Freestyle = 38
Breaststroke = 44
Backstroke = 41
Butterfly = 36

The total time for the coach's prediction is 159 seconds which is 2 minutes and 39 seconds.

The assignment was based on the Hungarian algorithm.
Team 1 Results:

Freestyle: Dylan
Breaststroke: Brandon
Backstroke: Cameron
Butterfly: Asher
∴
Freestyle: 28
Breaststroke: 44
Backstroke: 37
Butterfly: 34

The total time for the assignment that came from the Hungarian algorithm is 143 seconds which is 2 minutes and 23 seconds.

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The different between the prediction and the assignment for Team 1 was: $159 - 143 = 16$ There was a 16 second different between what the coach had predicted for the team and for what the best assignment for the team was.	While the prediction was based on which of the four students had the fastest time in each swim stroke, the Team 1 results were based on having a spread average across the four swim strokes. Overall looking at the two results the Team 1 results showed having an overall faster time for each swimming stroke. If the coach had to decide on whether to pick the prediction results or the T1 results so that the swimming team could have the best possible outcome at the interschool swimming carnival, the T1 result would be the right choice.

The coach decided that he wants to create a second relay team from the remaining five students, Ethan, Finn, George, Harry and Isaac. Other then making a prediction the coach decides to go straight and determine which stroke each student should swim using the Hungarian Algorithm. The coach called the new team T2

(*Original Matrix*)
This is the original matrix:

$$\text{Matrix T2} = \begin{bmatrix} 32 & 38 & 35 & 33 & 35 \\ 46 & 46 & 47 & 43 & 46 \\ 42 & 43 & 40 & 35 & 41 \\ 36 & 41 & 40 & 37 & 41 \end{bmatrix}$$

(*Making the matrix square*)
The T2 matrix has more columns than rows, so we add dummy rows with the numbers zero to make the matrix square:

$$\begin{matrix} 32 & 38 & 35 & 33 & 35 \\ 46 & 46 & 47 & 43 & 46 \\ 42 & 43 & 40 & 35 & 41 \\ 36 & 41 & 40 & 37 & 41 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

(*Subtracting row minimum*)
We subtract the row minimum from each row:

$$\begin{matrix} 0 & 6 & 3 & 1 & 3 & (-32) \\ 3 & 3 & 4 & 0 & 3 & (-43) \\ 7 & 8 & 5 & 0 & 6 & (-35) \\ 0 & 5 & 4 & 1 & 5 & (-36) \\ 0 & 0 & 0 & 0 & 0 & (-0) \end{matrix}$$

(*Subtracting column minimum*)
Because each column contains a zero, subtracting column minimum is not needed:

$$\begin{matrix} 0 & 6 & 3 & 1 & 3 \\ 3 & 3 & 4 & 0 & 3 \\ 7 & 8 & 5 & 0 & 6 \\ 0 & 5 & 4 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ (-0) & (-0) & (-0) & (-0) & (-0) \end{matrix}$$

(*Covering all the zeros with a minimum number of lines*)
3 lines are needed to cover all the zeros:

$$\begin{matrix} 0 & 6 & 3 & 1 & 3 \\ 3 & 3 & 4 & 0 & 3 \\ 7 & 8 & 5 & 0 & 6 \\ 0 & 5 & 4 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

(*Create additional zeros*)
The number of lines is smaller than the 5 lines needed. So, we subtract the smallest uncovered number which is 3 from all uncovered numbers and add it to all the number that are covered twice.

$$\begin{matrix} 0 & 3 & 0 & 1 & 0 \\ 3 & 0 & 1 & 0 & 0 \\ 7 & 5 & 2 & 0 & 3 \\ 0 & 2 & 1 & 1 & 2 \\ 3 & 0 & 0 & 3 & 0 \end{matrix}$$

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(*Covering all the zeros with a minimum number of lines*)

5 line are needed to cover all zeros:

$$\begin{matrix} 0 & 3 & 0 & 1 & 0 \\ 3 & 0 & 1 & 0 & 0 \\ 7 & 5 & 2 & 0 & 3 \\ 0 & 2 & 1 & 1 & 2 \\ 3 & 0 & 0 & 3 & 0 \end{matrix}$$

(*The assignment*)

Because there are 5 lines needed, the zeros show an assignment:

$$\text{Matrix T2} = \begin{matrix} 0 & 3 & 0 & 1 & 0 \\ 3 & 0 & 1 & 0 & 0 \\ 7 & 5 & 2 & 0 & 3 \\ 0 & 2 & 1 & 1 & 2 \\ 3 & 0 & 0 & 3 & 0 \end{matrix}$$

The second team was based on the five other student who didn't make team 1, the results for team 2 were:

Freestyle = Isaac
Breaststroke Finn
Backstroke = Harry
Butterfly = Ethan
Reserve = George
∴
Freestyle = 35
Breaststroke = 46
Backstroke = 35
Butterfly = 36

The total time for this team to complete the relay is:
$35 + 46 + 35 + 36 = 152$

The total time for the assignment that came from the Hungarian algorithm for Team 2 is 152 seconds which is 2 minutes and 32 seconds.

The different between the assignment for Team 1 and the assignment for Team 2 was:

$152 - 143 = 9$

There was a 9 second different between what the first assignment for Team 1 was and the second assignment for Team 2.

The reason behind the coach creating a second team was to see if the second team was faster than the first team. Looking at both result Team 2's total time was slower than Team 1's total time by 9 second.

If the coach had to choose between Team 1 and Team 2 the best optimal decision would be to choose Team 1 as the school has the best chance of winning with Team 1.

After creating Team 1 and Team 2 for coach decide to try to make a better Team 1 then the T1 found in the four students. The coach decides to use all nine students to determine a new Team 1 and Team 2.

(*Original Matrix*)

This is the original matrix:

Matrix F1 =

$$\begin{bmatrix} 34 & 35 & 38 & 28 & 32 & 38 & 35 & 33 & 35 \\ 41 & 44 & 45 & 45 & 46 & 46 & 47 & 43 & 46 \\ 41 & 43 & 37 & 42 & 42 & 43 & 40 & 35 & 41 \\ 43 & 38 & 38 & 36 & 36 & 41 & 40 & 37 & 41 \end{bmatrix}$$

(*Making the matrix square*)

The F1 matrix has more columns than rows, so we add dummy rows with the numbers zero to make the matrix square:

$$\begin{matrix} 34 & 35 & 38 & 28 & 32 & 38 & 35 & 33 & 35 \\ 41 & 44 & 45 & 45 & 46 & 46 & 47 & 43 & 46 \\ 41 & 43 & 37 & 42 & 42 & 43 & 40 & 35 & 41 \\ 34 & 38 & 38 & 36 & 36 & 41 & 40 & 37 & 41 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

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(*Subtracting rows minimum*)
We subtract the row minimum from each row:

6	7	10	0	4	10	7	5	7	(−28)
0	3	4	4	5	5	6	2	5	(−41)
6	8	2	7	7	8	5	0	6	(−35)
0	4	4	2	2	7	6	3	7	(−34)
0	0	0	0	0	0	0	0	0	(−0)
0	0	0	0	0	0	0	0	0	(−0)
0	0	0	0	0	0	0	0	0	(−0)
0	0	0	0	0	0	0	0	0	(−0)
0	0	0	0	0	0	0	0	0	(−0)

(*Subtracting columns minimum*)
We subtract the row minimum from each row:

6	7	10	0	4	10	7	5	7
0	3	4	4	5	5	6	2	5
6	8	2	7	7	8	5	0	6
0	4	4	2	2	7	6	3	7
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
(−0)	(−0)	(−0)	(−0)	(−0)	(−0)	(−0)	(−0)	(−0)

(*Covering all the zeros with a minimum number of lines*)
8 line are needed to cover all the zeros:

6	7	10	0	4	10	7	5	7
0	3	4	4	5	5	6	2	5
6	8	2	7	7	8	5	0	6
0	4	4	2	2	7	6	3	7
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(*Create additional zeros*)
The number of lines is smaller than 9. The smallest uncovered number is 2. We subtract this number from all uncovered elements and add it to all elements that are covered twice:

8	7	10	0	4	10	7	5	7
0	1	2	2	3	3	4	0	3
8	8	2	7	7	8	5	0	6
0	2	2	0	0	5	4	1	5
2	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0

(*Covering all the zeros with a minimum number of lines*)
9 lines are needed to cover all the zeros:

8	7	10	0	4	10	7	5	7
0	1	2	2	3	3	4	0	3
8	8	2	7	7	8	5	0	6
0	2	2	0	0	5	4	1	5
2	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0

(*The assignment*)
Because there are 9 lines needed, the zeros show an assignment:

Matrix F1 =

8	7	10	**0**	4	10	7	5	7
0	1	2	2	3	3	4	0	3
8	8	2	7	7	8	5	**0**	6
0	2	2	0	**0**	5	4	1	5
2	**0**	0	0	0	0	0	0	0
2	0	**0**	0	0	0	0	0	0
2	0	0	0	0	**0**	0	0	0
2	0	0	0	0	0	**0**	0	0
2	0	0	0	0	0	0	0	**0**

(*New First Team Results*)
Freestyle: Dylan
Breaststroke: Asher
Backstroke: Harry
Butterfly: Ethan
∴
Freestyle: 28
Breaststroke: 41
Backstroke: 35
Butterfly: 36

(*New Second Team Results*)
Freestyle: Brandon
Breaststroke: Cameron
Backstroke: George
Butterfly: Finn
Reserve: Isaac
∴
Freestyle: 35
Breaststroke: 45
Backstroke: 40
Butterfly: 41

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The total time for the new First team to complete the relay is:

$28 + 41 + 35 + 36 = 140$

The total time for the assignment that came from the Hungarian algorithm for the new Team 1 is 140 seconds which is 2 minutes and 20 seconds.

The total time for the Second team to complete the relay is:

$35 + 45 + 40 + 41 = 161$

The total time for the assignment that came from the Hungarian algorithm for the new Team 2 is 161 seconds which is 2 minutes and 41 seconds.

The different between the assignment for new Team 1 and the assignment for new Team 2 is:

$161 - 140 = 21$

There was a 21 second different between what the assignment for the new Team 1 was and the second assignment for new Team 2 was.

Using all nine students score, the coach was able to create the best possible relay team for the interschool carnival. The first team that consist of, Dylan, Asher, Harry and then Ethan and the second time with Brandon, Cameron, George and then Finn with Isaac being a reserve for team 2. The total time it would take for team 1 to finish the relay is 140 seconds while team 2 would take 160 seconds. So, its in best interest to choose team 1 for the relay team.

During a soccer carnival on the weekend before the interschool swimming carnival, George broke his left leg and had to pull out of the roster for the relay team. Because of this implication the coach decided to work out Team 1 and Team 2 again from the remaining eight student to determine if they would be any change because of Georges absent.

(*Original matrix*)

This is the original matrix:

Matrix F2 =

$$\begin{bmatrix} 34 & 35 & 38 & 28 & 32 & 38 & 33 & 35 \\ 41 & 44 & 45 & 45 & 46 & 46 & 42 & 46 \\ 41 & 43 & 37 & 42 & 42 & 43 & 45 & 41 \\ 34 & 38 & 38 & 36 & 36 & 41 & 37 & 41 \end{bmatrix}$$

(*Making the matrix square*)

The F2 matrix has more columns than rows, so we add dummy rows with the numbers zero to make the matrix square:

$$\begin{matrix} 34 & 35 & 38 & 28 & 32 & 38 & 33 & 35 \\ 41 & 44 & 45 & 45 & 46 & 46 & 43 & 46 \\ 41 & 43 & 37 & 42 & 42 & 43 & 45 & 41 \\ 34 & 38 & 38 & 36 & 36 & 41 & 37 & 41 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

(*Subtracting rows minimum*)

We subtract the row minimum from each row:

$$\begin{matrix} 6 & 7 & 10 & 0 & 4 & 10 & 5 & 7 & (-28) \\ 0 & 3 & 4 & 4 & 5 & 5 & 2 & 5 & (-41) \\ 6 & 8 & 2 & 7 & 7 & 8 & 0 & 6 & (-35) \\ 0 & 4 & 4 & 2 & 2 & 7 & 3 & 7 & (-34) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (-0) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (-0) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (-0) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (-0) \end{matrix}$$

(*Subtracting columns minimum*)

Because each column has a zero, subtracting column minimum is not needed.

$$\begin{matrix} 6 & 7 & 10 & 0 & 4 & 10 & 5 & 7 \\ 0 & 3 & 4 & 4 & 5 & 5 & 2 & 5 \\ 6 & 8 & 2 & 7 & 7 & 8 & 0 & 6 \\ 0 & 4 & 4 & 2 & 2 & 7 & 3 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ (-0) & (-0) & (-0) & (-0) & (-0) & (-0) & (-0) & (-0) \end{matrix}$$

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(*Covering all the zeros with a minimum number of lines*)
7 lines are needed to cover all the zeros:

6	7	10	0	4	10	5	7
0	3	4	4	5	5	2	5
6	8	2	7	7	8	0	6
0	4	4	2	2	7	3	7
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

(*Create additional zeros*)
The number of lines is smaller than the 8 lines needed. So, we subtract the smallest uncovered number which is 2 from all uncovered numbers and add it to all the number that are covered twice.

8	7	10	0	4	10	5	7
0	1	2	2	3	3	0	3
8	8	2	7	7	8	0	6
0	2	2	0	0	5	1	5
2	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0

(*Covering all the zeros with a minimum number of lines*)
8 lines are needed to cover all the zeros:

8	7	10	0	4	10	5	7
0	1	2	2	3	3	0	3
8	8	2	7	7	8	0	6
0	2	2	0	0	5	1	5
2	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0

(*The assignment*)
Because there are 8 lines needed, the zeros show an optimal assignment:

Matrix F2 =

8	7	10	0	4	10	5	7
0	1	2	2	3	3	0	3
8	8	2	7	7	8	0	6
0	2	2	0	0	5	1	5
2	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0

(*F2 – Team 1 Results*)
Freestyle: Dylan
Breaststroke: Asher
Backstroke: Harry
Butterfly: Ethan
∴
Freestyle: 28
Breaststroke: 41
Backstroke: 35
Butterfly: 36

(*F2 – Team Result*)
Freestyle: Brandon
Breaststroke: Cameron
Backstroke: Finn
Butterfly: Isaac
∴
Freestyle: 35
Breaststroke: 45
Backstroke: 43
Butterfly: 41

The total time for the new First team to complete the relay is:
$28 + 41 + 35 + 36 = 140$

The total time for the assignment that came from the Hungarian algorithm for the new Team 1 is 140 seconds which is 2 minutes and 20 seconds.

The total time for the new Second team to complete the relay is:
$35 + 45 + 43 + 41 = 164$

The total time for the assignment that came from the Hungarian algorithm for the new Team 1 is 164 seconds which is 2 minutes and 44 seconds.

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THE HUNGARIAN ALGORITHM

The different between the assignment for new Team 1 and the assignment for new Team 2 is: $164 - 140 = 24$ There was a 24 second different between what the assignment for the new Team 1 was and the second assignment for new Team 2 was.	Because of George broke his leg during the soccer carnival, the coach must re-calculate the relay team using the 8 students who volunteer. The first team again consisted of Dylan, Asher, Harry and then Ethan while the second team now consisted of Brandon, Cameron, Finn and then Isaac. Because of George's absent there was no change to team 1 as George never made it to team 1 but with team 2, instead of the team taking 161 second to complete the relay it would now take the team 164 second to complete the relay. Because of the increase in time, the best choice would be to choose Relay Team 1 to compete in the swimming carnival.

There were many limitations that could of change the whole outcome to the result of the swimming carnival. Starting with the coaches first choice to choose the first team out of the first four students. If the coaches decide to go with any four students for team 1 could the result in the end change. This could be something to go on and see if changing around the order would affect anything. Another limitation was which student broke their leg, if the student who broke their leg was part of Team 1 which of the student from Team 2 would have to move up and would that change the coach's decision to choose Team 1 over Team 2. A limitation that could would occur more than other with carnival is, if a student or more than one student is sick of the day of the carnival, the coach would need to calculate another team with the remaining students, or would a reserve just be moved up. There are many limitations towards this investigation as many things could go wrong and taking out the fastest student in one stroke could change the whole result.

During the first part of the investigation the coach decide to choose the first four students who sign up for the interschool swimming carnival, Asher, Brandon, Cameron and Dylan. The coach decides to use the Hungarian algorithm to solve which student should swim in each stroke. The coach predicted that for each of the four student who ever was the fastest in each stroke would swim that stroke. The prediction was that Cameron would do Freestyle, Brandon would do Breaststroke, Asher would do Backstroke and Dylan would do Butterfly. The coach summed up that it would take the team 159 second to complete the relay. After calculating the result using the algorithm, the result was that Dylan would do Freestyle, Brandon would do Breaststroke, Cameron would do Backstroke and Asher would do Butterfly. The total time it would take for this team to complete the relay is 143 seconds, 16 second fastest than what the coach first predicated. Now knowing that the algorithm gave the best possible team, the coach decides to calculate the five remaining students to see if their team would beat the first team that was created. The results were that Isaac would do Freestyle, Finn would do Breaststroke, Harry would Backstroke, and Ethan would do Butterfly, while George would be a reserve as only four students could be in the relay. The total time for the new team or team 2 to finish the relay was 152 seconds, 9 seconds slower than the first team. With this new information the coach decides on team 1 again but wanted to see what the outcome was if instead of splitting the student up into two group, he would use all nine students to calculate the best possible two team and then compare them to see which is faster. The first team that is create consist that Dylan would do Freestyle, Asher would do Breaststroke, Harry would do Backstroke and Ethan would do Butterfly. The second team consists that Brandon would do Freestyle, Cameron would do Breaststroke, George would do Backstroke and Finn would do Butterfly. The first teams total time to finish the relay was 140 second, 3 second faster than the first team that the coach made when he chooses the four students at the start. While the second team got slower with a total time of 161 second, being 9 seconds slower than the first team 2. With team 1 having 140 second and team 2 having 161 second, team 1 is 21 second fastest now and will surely be picked as the new team to compete in the swimming carnival. But on a sudden notice George broke his leg at a soccer match just before the coach decide to choose the final team, so the coach decide to calculate the team 1 and 2 again with the 8 remaining students. As the result were calculated there was no change to the already fast team 1 as George wasn't in team 1 to begin with so team 1 was still the same, but team 2 change with the new team 2 being that Brandon would do Freestyle, Cameron would do Breaststroke, Finn would do Backstroke and Isaac who was the reserve now is doing Butterfly. But the total time for team 2 to finish the relay increase by 3 second and now it took the team 164 second to finish which is 24 second

slower than team 1 still with 140 second to finish the relay. Even with one student breaking their leg the clear team to be chosen is team 1.
With the interschool swimming carnival closing in for the vested city private school the principal of the school asks the swimming coach if he had made up his mind for which a team of student who would compete for the carnival. After all the calculation were made and the result were tested repeatedly, the clear team to compete for the 4 x 50 metre relay would be Team 1 with the students Dylan, Asher, Harry and Ethan, this team would give the best possible chance for winning the interschool swimming carnival.

Bibliography

www.hungarianalgorithm.com/solve.php for the method of the Hungarian algorithm.