Modelling with Linear Relationships – Practice Test

Stage 2 General Mathematics

Question 1: Bunnings sells sausages and cans of soft drink to draw in customers. Usually a sausage costs $2.50 and a can of soft drink costs $2.00. They are offering deals aimed at families as more families entering Bunnings tends to increase their sales. In their 'Family Deal 1' they offer 4 sausages and 4 soft drinks for $12.00. In their 'Family Deal 2' they offer 9 sausages and 5 cans of soft drink for $23.

Is it cheaper to buy one of the special deals, and how much do the sausages and cans of soft drink cost in the family bundles? /4

normal cost $s = 2.50$ $d = 2.00$

Family Deal 1 $4s + 4d = 12.00$ $(\times 5)$ multiply to create
Family Deal 2 $9s + 5d = 23.00$ $(\times 4)$ an unknown with the same coefficient

① $20s + 20d = 60$
② $36s + 20d = 92$

② − ①
$16s = 32$ ($\div 16$ b.s.)
$s = 2$

substitute $s = 2$ into $4s + 4d = 12$
$4s + 4d = 12$
$(4 \times 2) + 4d = 12$
$8 + 4d = 12$ (-8 b.s.)
$4d = 4$ ($\div 4$ b.s.)
$d = 1$

It is cheaper to buy one of the bundles as sausages only cost $1 and drinks cost $1 which are both cheaper than cost price.

Bunnings largest competitor is offering 13 sausages and 10 soft drinks for $45. Is it better to buy in bulk from the competitor? Provide calculations and justify. /3

Calculate cost of $13s + 10d$ with prices previously calculated

Cost at Bunnings $(13 \times 2) + (10 \times 1) = \$36$

It is better to buy in bulk from bunnings as it will still be cheaper than the competitor ($36 < $45).

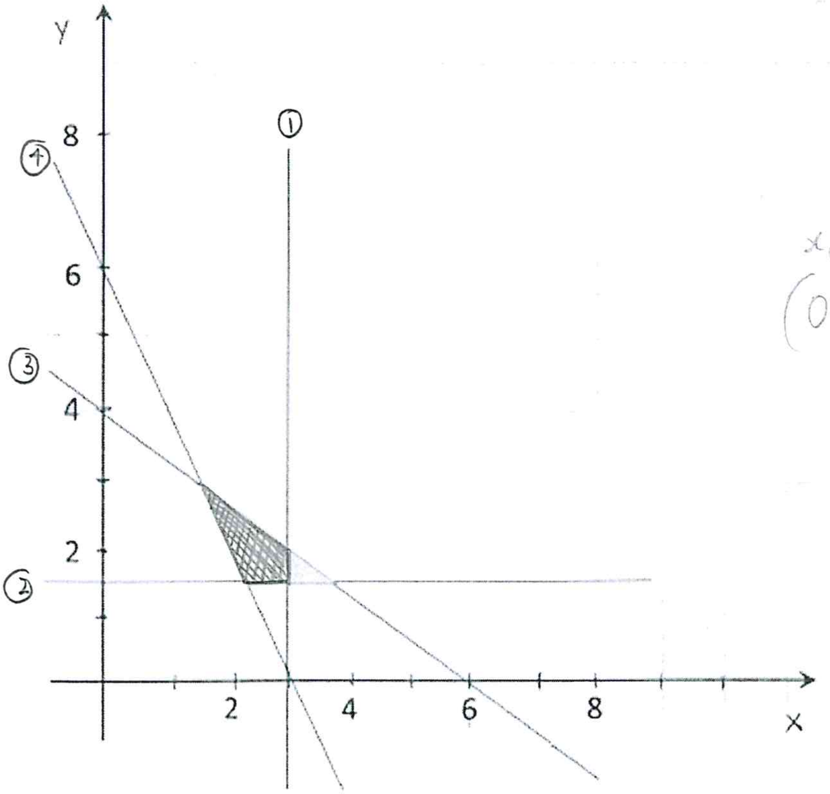
Question 2: In a money box there are 71 coins. Some are 20¢ coins and the rest are 10¢ coins. Their total value is $8.30. Find the number of each type of coin. /5

$x + y = 71$ rearrange $y = 71 - x$ ①
$10x + 20y = 830$ ②

sub ① → ②
$10x + 20(71 - x) = 830$
$10x + 1420 - 20x = 830$
$-10x + 1420 = 830$ (−1420 b.s.)
$-10x = -590$ (÷ −10 b.s.)
$x = 59$

sub $x = 59$ into $x + y = 71$
$x + y = 71$
$59 + y = 71$ (−59 b.s.)
$y = 12$
There are 59 10¢ coins and 12 20¢ coins

b) Using the graph below, write down the inequalities that mark the boundaries for the feasible region. /6



$x_1\ y_1\quad x_2\ y_2$
$(0, 6)\ (3, 0)$

① $\boxed{x \leq 3}$ as only shaded below the line
② $\boxed{y \geq 1.5}$ as only shaded above the line.

③ Option ①
Use x and y intercept method in reverse
x int = 6 y int = 4
When $y = 0$ $x = 6$, when $x = 0$ $y = 4$
∴ $2x + 3y = 12$ shaded below
$\boxed{2x + 3y \leq 12}$

Option ② $y = mx + c$ method
$y = mx + c$ $c = 4$ as y intercept
$y = mx + 4$ m = slope
slope $= \frac{y_2 - y_1}{x_2 - x_1}$
$y = -\frac{4}{6}x + 4$
slope $= \frac{0 - 4}{6 - 0}$
$\boxed{y \leq -\frac{4}{6}x + 4}$
slope $= -\frac{4}{6}$

④ $2x + y = 6$
shaded above
∴ $\boxed{2x + y \geq 6}$

OR

$y = mx + c$ $m = \frac{0 - 6}{3 - 0}$
$y = mx + 6$ $m = -\frac{6}{3}$
$y = -2x + 6$ $= -2$
$y \geq -2x + 6$

Question3: Gerald the alpaca needs to eat at least 7kg of food a day. His diet is made of pellets and grass. The pellets contain 3 units of protein and 3 units of carbohydrates per kilogram whereas grass contains 3 units of protein and 1 unit of carbohydrates per kilogram. Gerald requires at least 18 units of carbohydrates but no more than 33 units of protein.

a) Complete the table below and write the constraints that represent this information.

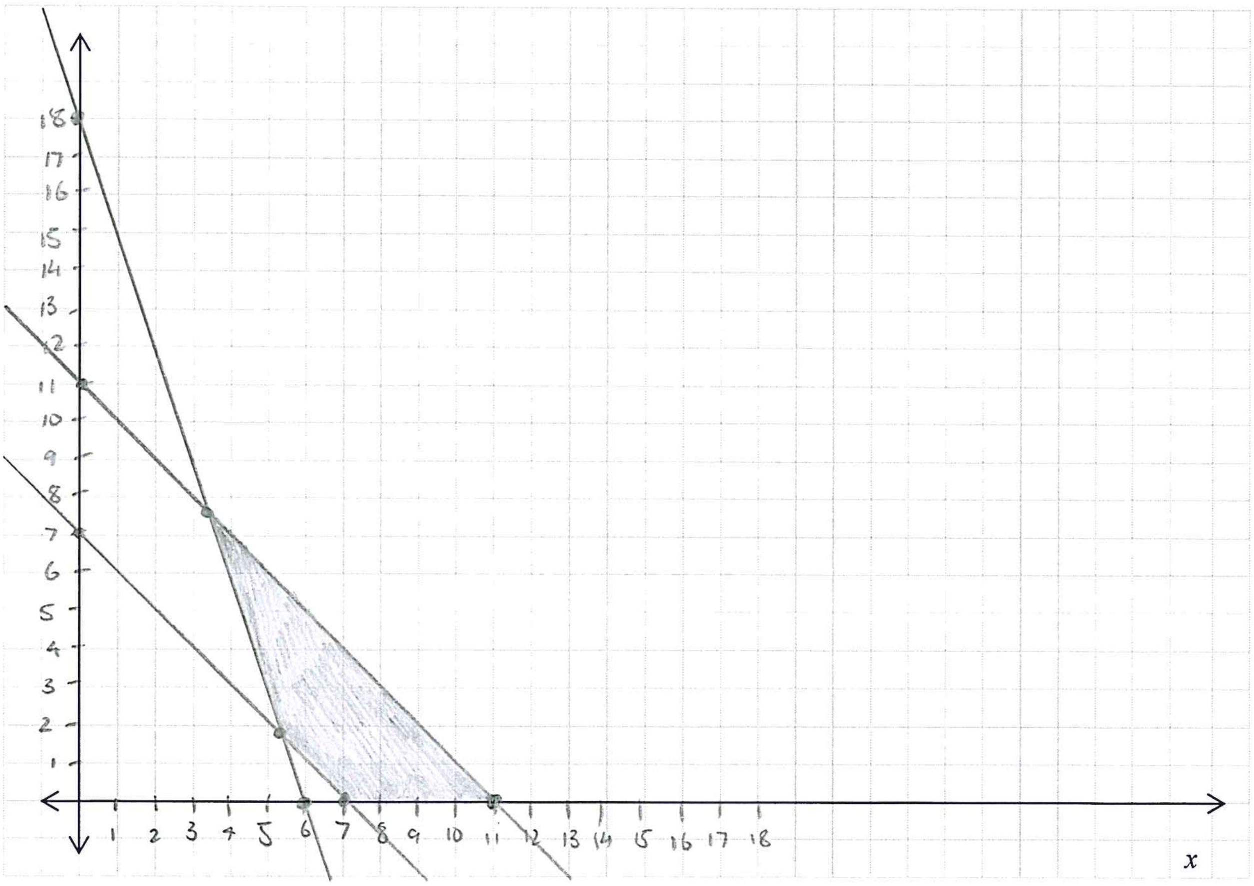
/4

Type	Carbohydrates	Protein	
Pellets	3	3	x
Grass	1	3	y
At most/or least	≥ 18	≤ 33	≥ 7

$x + y \geq 7$
$3x + y \geq 18$
$3x + 3y \leq 33$
$x \geq 0$
$y \geq 0$

b) Draw the constraints on the graph below and label them (including x≥0 and y≥0). Shade the feasible region.

/5



c) If pellets cost $8.00 per kilogram and grass costs $4.00 per kilogram, write an equation that shows the calculation of cost and find the minimum cost to feed Gerald. /4

minimise $8x + 4y$

Vertex Coordinate	Cost $8x+4y$
(7, 0)	56
(11, 0)	88
(5·5, 1·5)	50 ← Minimum
(3·5, 7·5)	58

To minimise cost Gerald should be fed 5·5kg of pellets and 1·5kg of grass

d) Why can't I just buy Gerald 6kg of ~~grain~~ pellets and 6kg of grass per day? /1

Carbs = 24 Protein = 36 if fed 6kg of pellets and 6kg of grass.
This would meet the carbohydrate requirement but exceeds the protein constraint.

c) Due to scarcity of grass after the bushfires, its price doubled to $8.00 per kilogram.

i) What effect has been made on the minimum cost due to this price rise? Show calculations to justify your answer. /5

Now minimising $8x + 8y$

Vertex Coordinate	Cost $8x + 8y$
(7, 0)	56 ←
(11, 0)	88
(5.5, 1.5)	56 ←
(3.5, 7.5)	88

Minimum

With the change in price Gerald could still be fed 5.5kg of pellets with 1.5kg of grass OR 7kg of pellets only.

Question 4: Each week a cat needs at least 225 units of carbohydrate, 80 units of protein, and 90 units of fat. Two tins of cat food are analysed to establish their content. Tin A contains 25 units of carbohydrate, 10 units of protein, and 15 units of fat. Tin B contains 50 units of carbohydrate, 10 units of protein, and 9 units of fat. Tin A costs $6 and tin B costs $3.

a) Represent all the above food data in the table below. /4

Type	Carb	Protein	Fat	
A	25	10	15	x
B	50	10	9	y
At most/or least	≥ 225	≥ 80	≥ 90	

$25x + 50y \geq 225 \quad = \quad 5x + 10y \geq 45$

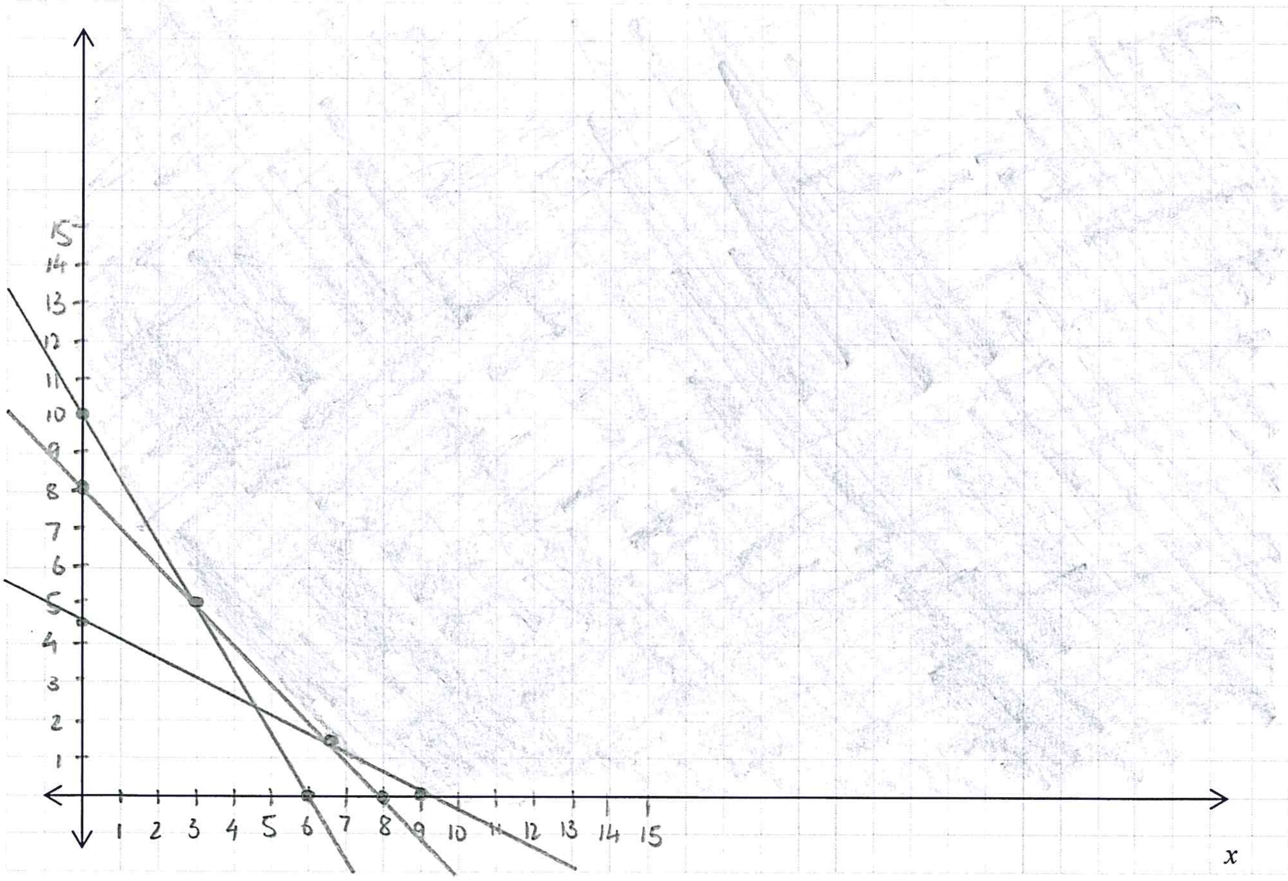
$10x + 10y \geq 80 \quad = \quad x + y \geq 8$

$15x + 9y \geq 90$

$x \geq 0$

$y \geq 0$

b) Graph the feasible region of the tabulated data that you have produced. /5



c) What is the objective function, and the minimum cost of the combination of tins which provide the necessary nutrients? Show all working /6

minimise $6x + 3y$

Vertex Coordinates	Cost $6x + 3y$
(0, 10)	30
(3,5)	33
(7,1)	45
(9,0)	54

← minimum

To minimise cost the cat should be fed 10 tins of tin B which will provide 90 units of fat, 500 units of carbohydrate and 100 units of protein which meets all requirements