STAGE 2 GENERAL MATHEMATICS

ASSESSMENT TYPE 1: SKILLS AND APPLICATIONS TASKS

TOPIC 2: MODELLING WITH MATRICES

Purpose

To demonstrate your ability to:

- understand mathematical concepts and relationships from within Topic 2: Modelling with Matrices
- select and apply mathematical techniques and algorithms to find solutions to problems
- interpret results, draw conclusions, and consider the reasonableness of solutions in context
- communicate mathematically and present mathematical information.

This assessment allows you to show your skills in understanding and appropriate use of the mathematical concepts, process, and strategies in the following:

(a) Subtopic 1.1: Application of matrices to network problems
(b) Subtopic 1.2: Application of matrices to transition problems

Assessment Conditions

This is a supervised assessment.

This task is conducted in a 90 minute lesson.

Assessment Design Criteria

Concepts and Techniques

CT 1 Knowledge and understanding of concepts and relationships.
CT 2 Selection and application of mathematical techniques and algorithms to find solutions to problems in a variety of contexts.
CT 4 Use of electronic technology to find solutions to mathematical problems.

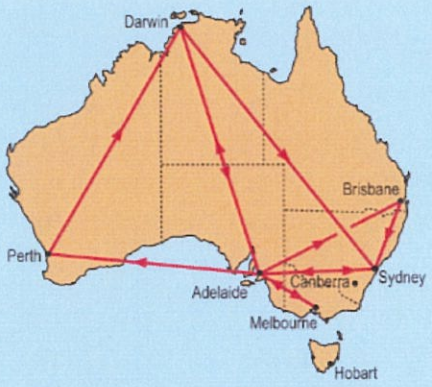
Reasoning and Communication

RC 1 Interpretation of mathematical results.
RC 2 Drawing conclusions from mathematical results with an understanding of their reasonableness and limitations.
RC 3 Use of appropriate notations representations and terminology.
RC 4 Communication of mathematical ideas and reasoning to develop logical arguments.

TOPIC 2: MODELLING WITH MATRICES

QUESTION 1:

A new airline, Auzzie Blu, has the flight paths shown on the map alongside.



a Draw a network to represent the situation.

b What is represented by:

i the nodes **ii** the edges?

c How many different flights does Auzzie Blu offer?

d Find the connectivity matrix **C** for Auzzie Blu flights.

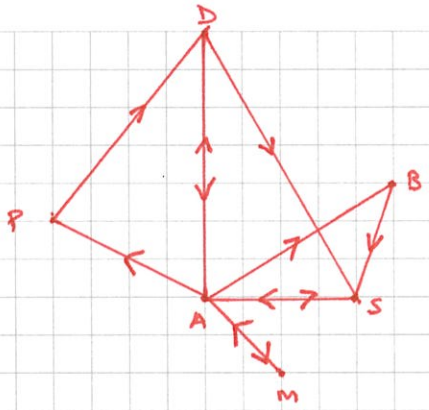
e Use matrix methods to determine:

i how many journeys between two cities cannot be made using at most one stopover

ii the maximum number of flights needed to travel between any two cities on the network.

f Auzzie Blu are considering a service upgrade by either making the flight from Adelaide to Perth "two-way", or by creating a Perth to Brisbane link. Investigate the effects that these proposed changes would have on your answers to **e**, and hence advise which would be the better decision.

(a)



b). nodes represent capital cities

. edges represent directional flight paths.

c) 11 different flights

d) $C =$

	A	B	D	M	P	S
A	0	1	1	1	1	1
B	0	0	0	0	0	1
D	1	0	0	0	0	1
M	1	0	0	0	0	0
P	0	0	1	0	0	0
S	1	0	0	0	0	0

(e)(i) 5 Journeys

- P to B
- P to M
- B to D
- B to M
- B to P

(ii) Perth to Brisbane
PDAB (3 flights)

(f) making Adelaide to Perth two way would reduce (e)(i) to 3

making Perth to Brisbane one way would only reduce (e)(i) to 4

∴ making Adelaide to Perth two way would be a better decision.

QUESTION 2:

In the reality television show "Stranded", five participants Amy, Derek, Hubert, Lee, and Mindy are left on a desert island for a month. Over the month, the viewers notice that:

Amy has influence over Lee
Derek has influence over Mindy
Hubert has influence over Amy and Derek
Lee has influence over Hubert
Mindy has influence over Hubert and Lee.

a Draw a network to illustrate these influences.

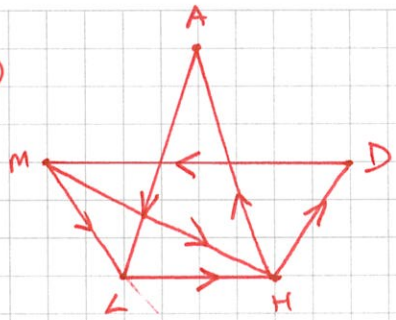
b Find the dominance matrix **D**, and hence find $\mathbf{D}^2$.

c **i** Which participants have the most *direct* influence over others?

ii Who has the most *indirect* influence?

d Find the supremacy matrix $\mathbf{S} = \mathbf{D} + \frac{1}{2}\mathbf{D}^2$, and use it to rank the participants according to influence.

(a)



(b)

$$D = \begin{array}{c} \\ A \\ D \\ H \\ L \\ M \end{array} \begin{array}{c} \begin{array}{ccccc} A & D & H & L & M \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 1 \\ 2 \\ 1 \\ 2 \end{array}$$

$$D^2 = \begin{array}{c} \\ A \\ D \\ H \\ L \\ M \end{array} \begin{array}{c} \begin{array}{ccccc} A & D & H & L & M \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 2 \\ 2 \\ 3 \end{array}$$

(c)(i) Mindy and Hubert (direct influence)

(ii) Mindy has indirect influence over 2 people (Amy and Derek).

(d) $S = D + \frac{1}{2}D^2$

$$S = \begin{bmatrix} 0 & 0 & \frac{1}{2} & 1 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 1 \\ 1 & 1 & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 1\frac{1}{2} & 1 & 0 \end{bmatrix} \begin{array}{c} 1\frac{1}{2} \\ 2 \\ 3 \\ 2 \\ 3\frac{1}{2} \end{array}$$

Ranking based on Supremacy matrix $S = D + \frac{1}{2}D^2$

Mindy, Hubert, Lee, Derek, Amy

QUESTION 3:

A university student is researching the connection between adults who smoke and the smoking habits of their children. Her results are presented in the table alongside.

		Next generation	
		Smoker	Non-smoker
This generation	Smoker	55%	45%
	Non-smoker	5%	95%

a Construct a transition matrix $\mathbf{T}_1$, with elements in decimal form.

b In the matrix $\mathbf{T}_1$, state the meaning of the value in row 1, column 2.

c Two generations ago, 60% of adults smoked. What percentage of adults smoke now?

d Find the steady state proportions.

e Due to extensive advertising campaigns highlighting the dangers of smoking, only 20% of smokers' children now take up smoking. Find the new transition matrix $\mathbf{T}_2$, and determine the new steady state proportions.

a)

$$T_1 = \begin{bmatrix} 0.55 & 0.45 \\ 0.05 & 0.95 \end{bmatrix}$$

b) 0.45 ie 45% of the children of smokers will be non-smokers

c) Two generations ago.

$$\begin{bmatrix} .60 & .40 \end{bmatrix} \begin{bmatrix} 0.55 & 0.45 \\ 0.05 & 0.95 \end{bmatrix}^2$$

$$= \begin{bmatrix} 0.225 & 0.775 \end{bmatrix}$$

$$= 22.5\%$$

d) steady state

$$\begin{bmatrix} .60 & .40 \end{bmatrix} \begin{bmatrix} 0.55 & 0.45 \\ 0.05 & 0.95 \end{bmatrix}^{12}$$

$$= \begin{bmatrix} .10 & .90 \end{bmatrix}$$

ie 10% will smoke

90% will not.

e)

$$T_2 = \begin{bmatrix} .20 & .80 \\ 0.05 & 0.95 \end{bmatrix}$$

$$T_{12} = \begin{bmatrix} .20 & .80 \\ .05 & .95 \end{bmatrix}^{12}$$

$$= \begin{bmatrix} 0.588 & 0.941 \\ 0.588 & 0.941 \end{bmatrix}$$

ie 5.88% will smoke 94.1% will not.

QUESTION 4:

In their weekly art class, students can choose between painting, pottery, and photography. The students change activities according to the table alongside.

This week \ Next week	Painting	Pottery	Photography
Painting	20%	15%	65%
Pottery	15%	15%	70%
Photography	20%	30%	50%

In the first week, 14 students chose painting, 12 students chose pottery, and 8 students chose photography.

a **i** Write down the initial state matrix **S** and the transition matrix **T**.

ii Interpret the value in row 2, column 1 of **T**.

b How many students are expected to participate in each activity:

i in the second week **ii** in the long term?

c Once the class sizes have reached their steady state, the art teacher notices the photography lab is too crowded. She decides that students will no longer be allowed to choose photography two weeks in a row.

i Explain why it is now impossible for the steady state proportion of students choosing photography to be greater than 50%.

ii 40% of students who choose photography this week now choose painting next week, and 60% choose pottery. Write down the new transition matrix, and hence find the new steady state of class sizes.

a (i) $S = \begin{bmatrix} Pa & Po & Ph \\ 14 & 12 & 8 \end{bmatrix}$ $T = \begin{bmatrix} 0.2 & 0.15 & 0.65 \\ 0.15 & 0.15 & 0.7 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}$

(ii) row 2, column 1 ie 15% of students who choose pottery this week will choose painting next week

b (i) $\begin{bmatrix} 14 & 12 & 8 \end{bmatrix} \begin{bmatrix} 0.2 & 0.15 & 0.65 \\ 0.15 & 0.15 & 0.7 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}$

6 choose painting
6 choose pottery
22 choose photography

(ii) $\begin{bmatrix} 14 & 12 & 8 \end{bmatrix} \begin{bmatrix} 0.2 & 0.15 & 0.65 \\ 0.15 & 0.15 & 0.7 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}^{52}$

6 choose painting
8 choose pottery
20 choose photography

C(i) If more than 50% choose photography one week, none of them can choose photography the next week, so less than 50% will choose photography the next week. Thus a steady state has not been reached.

(ii) $T = \begin{bmatrix} 0.2 & 0.15 & 0.65 \\ 0.15 & 0.15 & 0.70 \\ 0.4 & 0.6 & 0.0 \end{bmatrix}$

9 choose painting
11 choose pottery
14 choose photography

ST^{52}

Performance Standards for Stage 2 General Mathematics

	Concepts and Techniques	Reasoning and Communication
A	Comprehensive knowledge and understanding of concepts and relationships. Highly effective selection and application of mathematical techniques and algorithms to find efficient and accurate solutions to routine and complex problems in a variety of contexts. Successful development and application of mathematical models to find concise and accurate solutions. Appropriate and effective use of electronic technology to find accurate solutions to routine and complex problems.	Comprehensive interpretation of mathematical results in the context of the problem. Drawing logical conclusions from mathematical results, with a comprehensive understanding of their reasonableness and limitations. Proficient and accurate use of appropriate mathematical notation, representations, and terminology. Highly effective communication of mathematical ideas and reasoning to develop logical and concise arguments. Formation and testing of appropriate predictions, using sound mathematical evidence.
B	Some depth of knowledge and understanding of concepts and relationships. Mostly effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine and some complex problems in a variety of contexts. Attempted development and successful application of mathematical models to find mostly accurate solutions. Mostly appropriate and effective use of electronic technology to find mostly accurate solutions to routine and some complex problems.	Mostly appropriate interpretation of mathematical results in the context of the problem. Drawing mostly logical conclusions from mathematical results, with some depth of understanding of their reasonableness and limitations. Mostly accurate use of appropriate mathematical notation, representations, and terminology. Mostly effective communication of mathematical ideas and reasoning to develop mostly logical arguments. Formation and testing of mostly appropriate predictions, using some mathematical evidence.
C	Generally competent knowledge and understanding of concepts and relationships. Generally effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine problems in different contexts. Application of mathematical models to find generally accurate solutions. Generally appropriate and effective use of electronic technology to find mostly accurate solutions to routine problems.	Generally appropriate interpretation of mathematical results in the context of the problem. Drawing some logical conclusions from mathematical results, with some understanding of their reasonableness and limitations. Generally appropriate use of mathematical notation, representations, and terminology, with reasonable accuracy. Generally effective communication of mathematical ideas and reasoning to develop some logical arguments. Formation of an appropriate prediction and some attempt to test it using mathematical evidence.
D	Basic knowledge and some understanding of concepts and relationships. Some selection and application of mathematical techniques and algorithms to find some accurate solutions to routine problems in context. Some application of mathematical models to find some accurate or partially accurate solutions. Some appropriate use of electronic technology to find some accurate solutions to routine problems.	Some interpretation of mathematical results. Drawing some conclusions from mathematical results, with some awareness of their reasonableness. Some appropriate use of mathematical notation, representations, and terminology, with some accuracy. Some communication of mathematical ideas, with attempted reasoning and/or arguments. Attempted formation of a prediction with limited attempt to test it using mathematical evidence.
E	Limited knowledge or understanding of concepts and relationships. Attempted selection and limited application of mathematical techniques or algorithms, with limited accuracy in solving routine problems. Attempted application of mathematical models, with limited accuracy. Attempted use of electronic technology, with limited accuracy in solving routine problems.	Limited interpretation of mathematical results. Limited understanding of the meaning of mathematical results, their reasonableness or limitations. Limited use of appropriate mathematical notation, representations, or terminology, with limited accuracy. Attempted communication of mathematical ideas, with limited reasoning. Limited attempt to form or test a prediction.