Name:

HERITAGE COLLEGE

Semester 1 Examinations

2024

Year 9 Mathematics (Formative)

Time: 2 hours

QUESTION 1

Use the laws of indices to simplify the following:

(a) $x^3 \times x^{10}$

x^{3+10}

x^{13}

(b) $\frac{x^{12}}{x^3}$

x^{12-3}

x^9

(c) $10xy^3 \times 4y^4$

$40xy^7$

(d) $\frac{12x^5y^4}{3x^2y^3}$

$4x^3y$

(4 marks)

(e) $10x^0 - 1$

$10 \times 1 - 1$

$= 9$

(f) $\left(\frac{2x}{y^2}\right)^3$

$\frac{2^3x^3}{y^6} = \frac{8x^2}{y^6}$

(g) 3^{-4}

$\frac{1}{3^4}$

$= \frac{1}{81}$

(h) $\left(\frac{2}{7}\right)^{-2}$

$\frac{7^2}{4^2} = \frac{49}{16}$

Should be 4

(4 marks)

QUESTION 2

Write the following as ordinary decimal numbers:

(a) The length of the earth's equator is approximately 4.01×10^4 km

$40100 km$

(b) The wavelength of blue light is 4.75×10^{-7} m

$.000000475 m$

(2 marks)

QUESTION 3

Write the following using scientific notation:

(a) The probability of winning the lottery on Monday night is 0.000 000 141 62

1.4162×10^{-7}

(b) The distance from the Sun to Mars is about 249 000 000 km

$2.49 \times 10^8 km$

(2 marks)

QUESTION 4

Expand and simplify the following expressions using the appropriate rule:

(a) $3x(x-5)$

$3x^2 - 15x$

(b) $(x-5)(x-5)$

$x^2 - 10x + 25$

(4 marks)

(c) $(2x+7)(2x-7)$

$4x^2 - 14x + 14x - 49$

$4x^2 - 49$

(d) $(3x+4)(x-2)$

$3x^2 - 6x + 4x - 8$

$3x^2 - 2x - 8$

(4 marks)

QUESTION 5

(a) Find the perimeter P, of:

HINT: Add up all the side lengths

$(x+1)$ cm

$(2x+12)$ cm

(b) Find the shaded area A, of:

HINT: Area of a rectangle=Length×Width

x cm

$(x+2)$ cm

$3x$ cm

$(x+8)$ cm
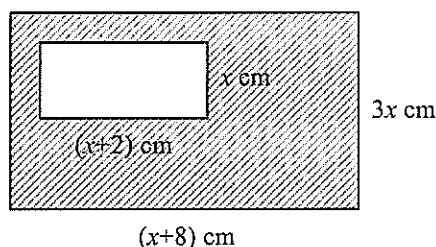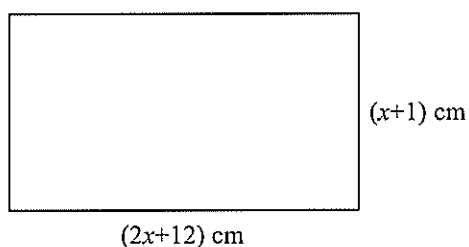
$2(x+1) + 2(2x+12)$

$2x+2 + 4x + 24$

$= 6x + 26$ cm

$3x(x+8) - x(x+2)$

$3x^2 + 24x - x^2 - 2x$

$2x^2 + 22x$

(4 marks)

QUESTION 6

Fully factorise the following

(a) $6x^2 - 2x$

$2x(3x - 1)$

(b) $2p(a + 7) + 3q(a + 7)$

$(2p + 3q)(a + 7)$

(4 marks)

(c) $cd - 5d + 3c - 15$

$d(c - 5) + 3(c - 5)$
$(d + 3)(c - 5)$

(d) $x^2 + 7x + 4x + 28$

$x(x + 7) + 4(x + 7)$
$(x + 4)(x + 7)$

(4 marks)

QUESTION 7

Solve the following linear equations:

(a) $\frac{x}{3} + 4 = 8$

$$\frac{x}{3} = 8 - 4$$

$$\frac{x}{3} = 4$$

$$x = 12$$

(b) $\frac{5x-3}{2} = 6$

$$5x - 3 = 12$$

$$5x = 15$$

$$x = 3$$

(4 marks)

(c) $9x + 5 = 5x - 3$

$$9x - 5x = -3 - 5$$

$$4x = -8$$

$$x = -2$$

(d) $\frac{3x-2}{7} = \frac{x+6}{4}$

$$4(3x-2) = 7(x+6)$$

$$12x - 8 = 7x + 42$$

$$12x - 7x = 42 + 8$$

$$5x = 50$$

$$x = 10$$

(4 marks)

QUESTION 8

The sum of three consecutive whole numbers is 66. What are the numbers? (Show a full algebraic solution.)

$$x + x + 1 + x + 2 = 66$$
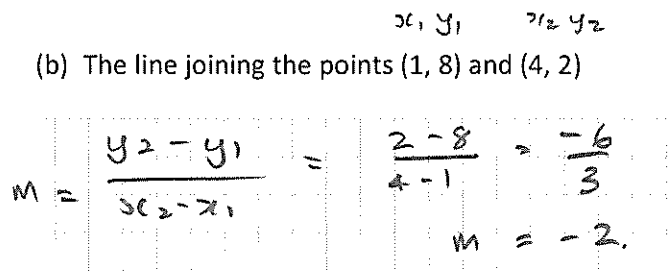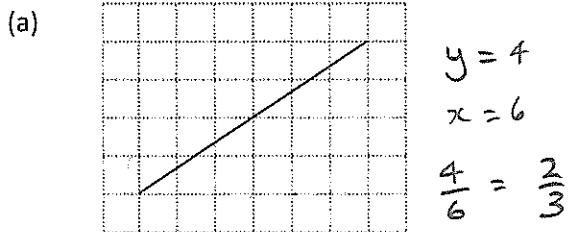
$$3x + 3 = 66$$

$$3x = 63$$

$$x = 21$$

21, 22, 23

(4 marks)

QUESTION 9

Calculate the gradient of the following lines:

x_1, y_1 x_2, y_2

(a)

$y = 4$

$x = 6$

$\dfrac{4}{6} = \dfrac{2}{3}$

(b) The line joining the points (1, 8) and (4, 2)

$$M = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 8}{4 - 1} = \frac{-6}{3}$$
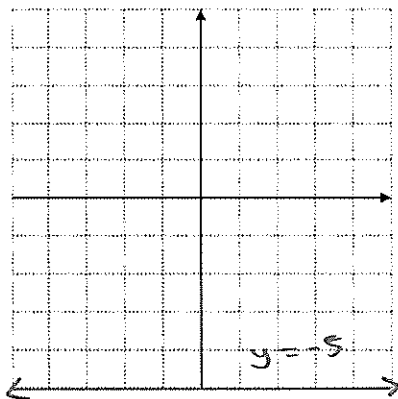
$$M = -2.$$

(2 marks)

QUESTION 10   Sketching Linear Graphs   (6 marks)
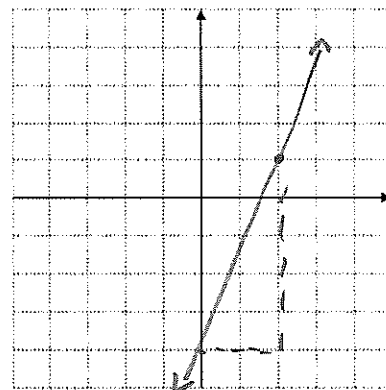
Sketch the graphs of the following lines: (show all reasoning)

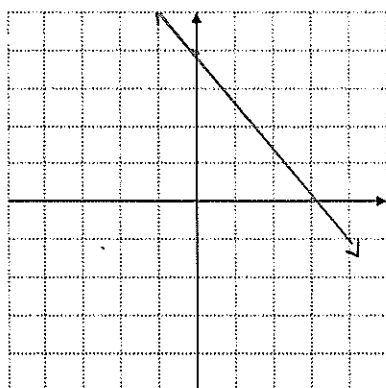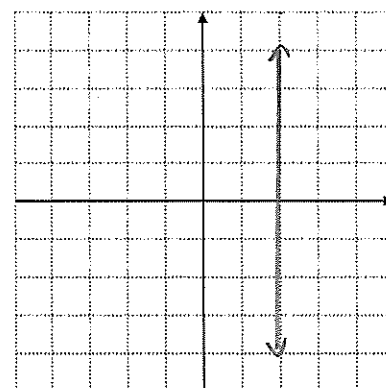(a) $y = -5$

(b) $y = \frac{5}{2}x - 4$

$y \text{ intercept} = -4$

$M = \dfrac{5}{2}$

$y = -5$

(c) $4x + 3y = 12$

$x = 0 \quad y = 4$

$x = 3 \quad y = 0$

(d) $x = 2$

QUESTION 11   Parallel and Perpendicular Lines   (5 marks)

Two lines are given by and $y = \frac{k}{5}x - 6$ ① and $4x - 3y - 2 = 0$ ②.

(a) Write down the gradient of the line represented by equation ①

$$\frac{k}{5}$$

(b) Rearrange equation ② to find its gradient

$$-3y = -4x + 2$$
$$y = \frac{-4}{-3}x + \frac{2}{3}$$
$$m = \frac{4}{3}x - \frac{2}{3}$$

(c) What is the connection between the gradients of parallel lines

The gradient is the same value. even though the y int may be diff.t

(d) Find the value of k such that the lines are parallel

$$\frac{k}{5} = \frac{4}{3} = \frac{20}{15}$$

QUESTION 12   Using Linear Relationships   (5 marks)

An electrician charges a fixed fee plus an hourly rate. A job that takes 2 hours to complete costs $280 and a job that takes 3.5 hours to complete costs $437.50

(a) Show that the linear model to express the charge, C, for t hours of work is given by $C = 105t + 70$

$C = 105t + 70$ ∴ if $t = 2$

$C = 105(2) + 70$

$C = \$280$

$t = 3.5$

$C = 105(3.5) + 70$

$C = \$437.50$

(b) Find the amount the electrician earns if a job takes 8 hours

$$C = 105(8) + 70$$
$$C = \$910.00$$

(c) Find the time taken to complete the job if the charge was $306.25

$C = 306.25$ rearrange

$306.25 = 105(t) + 70$

$306.25 - 70 = 105t$

$236.25 = 105t$

$t = \frac{236.25}{105}$

$t = 2.25$ or 2 hours 15 minutes.

QUESTION 13   Using the Null Factor Law   (4 marks)

Solve the following quadratic equations using factorising and the null factor law:

(a) $(x + 7)(x - 10) = 0$

$x = -7$ or $x = 10$

(b) $2x^2 - 6x = 0$

$2x(x - 3) = 0$

$x = 0$ or 3

QUESTION 14   Solving Quadratic Equations   (4 marks)

Solve the following quadratic equations using factorising and the null factor law:

(a) $(x + 8)(x - 2) = 6x$

Expand $x^2 - 2x + 8x - 16 = 6x$

Simplify $x^2 + 6x - 16 = 6x$

Rearrange $x^2 + 6x - 6x - 16 = 0$

            $x^2 - 16 = 0$

Factorise $(x - 4)(x + 4) = 0$

            $x = 4$ or -4

(b) $x^2 - 20x + 36 = -64$

$x^2 - 20x + 36 + 64 = 0$

$x^2 - 20x + 100 = 0$

$(x - 10)(x - 10) = 0$

$x = 10$

QUESTION 15   Problem Solving with Quadratic Equations   (6 marks)

(a)   Use the FOIL rule to show that $(x - 8)(x + 13) = x^2 + 5x - 104$
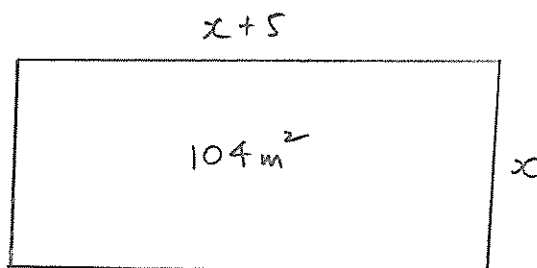
$x^2 + 13x - 8x - 104$ Expand brackets

$x^2 + 5x - 104$ Simplify

(b)   A rectangular plot has length 5 m more than its width. If its area is 104 m², find the dimensions of the plot. HINT: Part (a) should help you when it comes to factorising. (Show a full algebraic solution)

$x + 5$

$104 m^2$

x

$A = L \times W$

$104 = x(x + 5)$ Form an equation

$104 = x^2 + 5x$ Rearrange

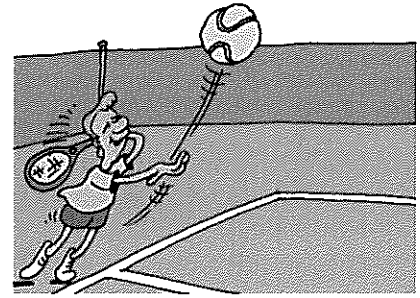$x^2 + 5x - 104 = 0$ Factorise

$(x - 8)(x + 13) = 0$

$x = 8$ or -13 reject as you cant have a negative width/length

QUESTION 16   Non-linear relationships   (7 marks)

Tennis player Kezia tosses the ball into the air before she serves it. The
ball's height above ground t seconds after it is tossed is given by the
function $H(t) = -5t^2 + 6t + 2$ metres.

(a)   Complete the table of values to plot the tennis ball's progress through the air.

t	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
H	2	2.55	3	3.35	3.6	3.75	3.8	3.75	3.6	3.35	3	2.55	2	1.35	0.6	-0.25
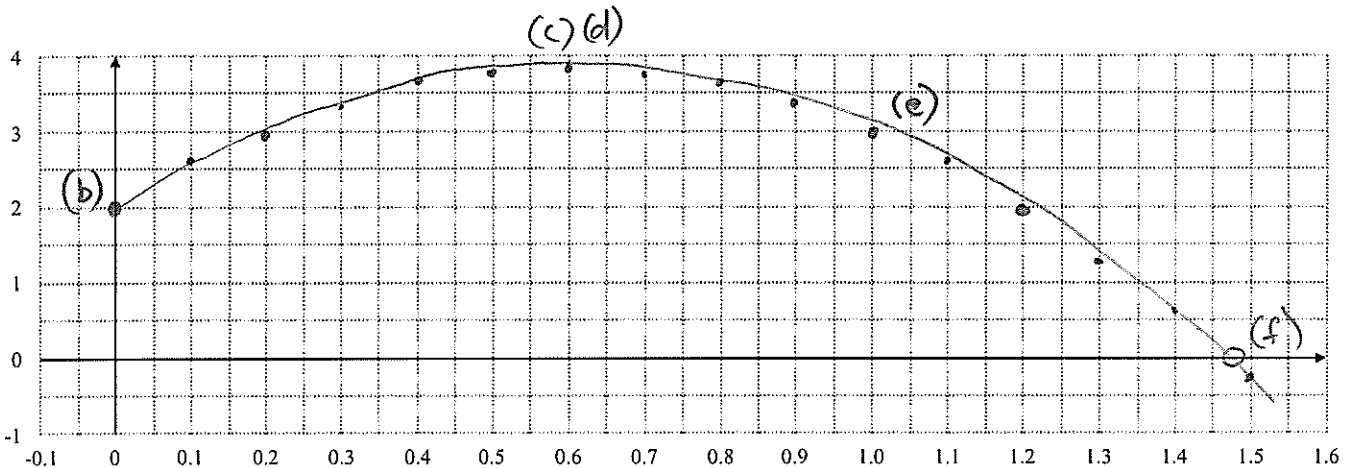
(b)   How high was the ball
          when it was released?

          $2\,m$

(c)   When did the tennis ball
          reach its maximum
          height?

          after 0·6 seconds

(d)   What was the maximum
          height reached by the
          tennis ball?

          $3·8\,m$

(e)   Kezia hits the ball when it is 3 m above the ground, and on its
          way down. How long after Kezia throws the ball up does she
          hit it?

          after 1·0 seconds

(f)   If Kezia missed the ball,
          estimate when it would hit
          the ground.

          after about
          1·47 seconds