South Australian
Certificate of Education

General Mathematics
2023

Question booklet

- Questions 1 to 8 (90 marks)
- Answer **all** questions
- Write your answers in this question booklet
- You may write on page 20 if you need more space

Examination information

Materials
- Question booklet
- SACE registration number label

Instructions
- Show appropriate working and steps of logic in this question booklet
- Use black or blue pen
- You may use a sharp dark pencil for diagrams and graphical representations
- Approved calculators may be used — complete the box below

Total time: 130 minutes
Total marks: 90

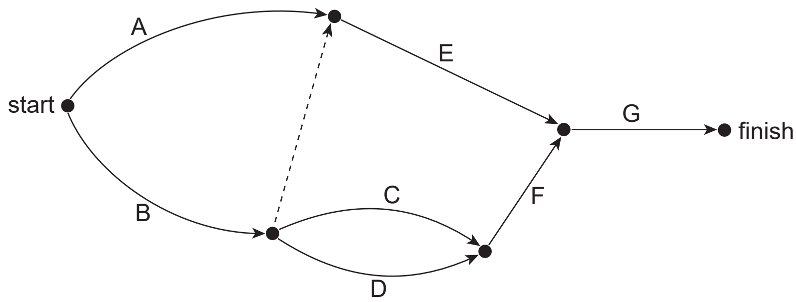
Attach your SACE registration number label here	Graphics calculator 1. Brand ________ Model ________ 2. Brand ________ Model ________

Government
of South Australia

Question 1 (8 marks)

A manufacturer recommends a number of steps to be followed to assemble a children's toy.

The sequence of steps required is shown in the network diagram below.



(a) Complete the precedence in Table 1 below for this network to show the prerequisites for tasks B and F.

Table 1

Task	*Prerequisites*
A	none
B	
C	B
D	B
E	A, B
F	
G	E, F

(2 marks)

(b) State a task that must always lie on the critical path for this network.

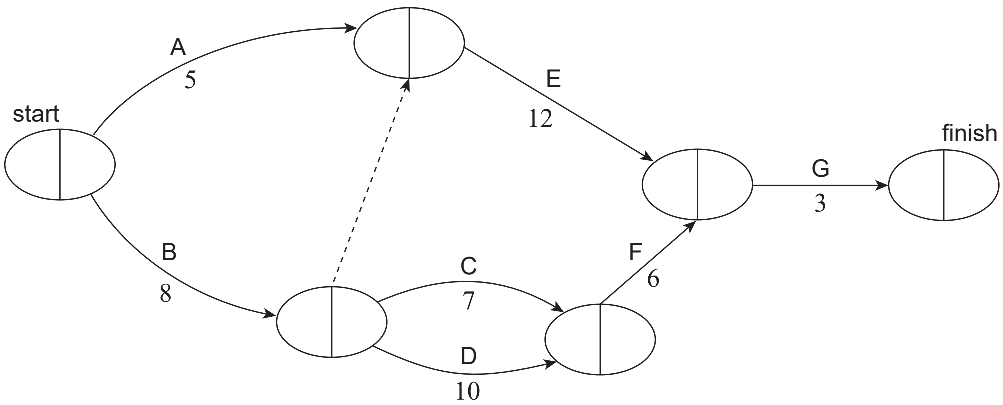
(1 mark)

(c) With reference to the network, explain why the dummy link is required.

(2 marks)

The network diagram below is repeated from page 2. In the version below, the completion time (in minutes) for each task is provided.

(d) *On this network diagram*, carry out a forward and backward scan.



(2 marks)

(e) State the minimum completion time for this network.

(1 mark)

 PLEASE TURN OVER

Question 2 (10 marks)

The waiting times for non-critical patients to be attended to in the emergency department of Hospital A are found to be normally distributed with a mean of 3.4 hours and a standard deviation of 1.1 hours.

(a) Calculate the probability of a non-critical patient having a waiting time between 3.4 hours and 4.5 hours.

(1 mark)

(b) In one month, 5500 people were attended to in the emergency department.

How many people would be expected to wait for more than 2 hours?

(2 marks)

(c) A study estimates that the 29% of non-critical patients who experience the longest waiting times are more likely to experience complications in their recovery.

Calculate the shortest waiting time for a patient who would be more likely to experience complications in their recovery.

(2 marks)

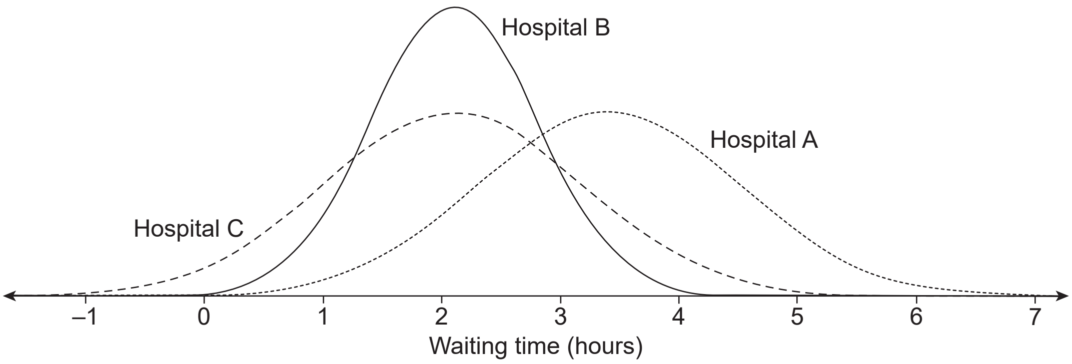
The waiting times for non-critical patients to be attended to in the emergency department of Hospital B are also found to be normally distributed with a mean of 2.1 hours and a standard deviation of 0.7 hours.

(d) Hospital B claims that 25% of non-critical patients will be attended to within 1.5 hours.

Using an appropriate calculation, *state* if this claim can be supported.

(2 marks)

The diagram below shows the probability distributions of the waiting times for non-critical patients to be attended to in the emergency departments of Hospitals A, B, and C.



(e) What are the mean and standard deviation for the waiting times for non-critical patients to be attended to in the emergency department of Hospital C? Tick the appropriate boxes to indicate your answers.

Mean (hours)	☐ 0.7	☐ 1.1	☐ 2.1	☐ 3.4
Standard deviation (hours)	☐ 0.7	☐ 1.1	☐ 2.1	☐ 3.4

(2 marks)

(f) From the probability distribution diagram above, the following statement can be made.

It is statistically possible for a non-critical patient to be attended to with less than zero hours of waiting time.

State why the statement is unreasonable.

(1 mark)

Question 3 (12 marks)

The management team of a sports club is considering two options, A and B, for borrowing $\$95\,000$ to conduct an immediate upgrade of the change rooms.

Option A

- An interest-only loan from a credit union for 8 years.
- Interest is charged at a flat rate of 6.8% per annum.
- This option also requires the club to make quarterly deposits into a sinking fund for 8 years.
- The sinking fund earns interest at 2.95% per annum, compounded quarterly.

(a) Calculate the yearly interest payment for the interest-only loan.

(1 mark)

(b) Calculate the quarterly payments that the club must make into the sinking fund.

(2 marks)

(c) (i) Show that the total cost of Option A for the club is approximately $\$136\,000$.

(1 mark)

(ii) State why the total cost stated in part (c)(i) is less than the amount that will be repaid to the credit union.

(1 mark)

Option B
- A reducing-balance loan from a bank for 8 years.
- Interest is charged at 5.75% per annum, compounded monthly.
- A monthly service charge of $\$25$ applies.

(d) (i) Calculate the comparison rate for Option B.

(3 marks)

(ii) Calculate the total cost of Option B for the club.

(1 mark)

(e) Suggest *one* reason why the club might not select the cheaper option when borrowing the $\$95\,000$.

(1 mark)

Inflation over this 8-year period is predicted to average 4.5% per annum.

(f) (i) Calculate the value of the $\$95\,000$ indexed for inflation in 8 years' time.

(1 mark)

(ii) State *one* limitation of waiting for 8 years to upgrade the change rooms, rather than doing the upgrade immediately.

(1 mark)

Question 4 (11 marks)

Table 2 below shows the number of long-footed potoroos during a 5-year breeding program.

Table 2: Number of long-footed potoroos during a breeding program (N)

Time in months (t)	0	6	12	18	24	30	36	42	48	54	60
Number of long-footed potoroos (N)	80	91	100	119	134	136	156	151	175	196	199

(a) (i) Using a linear model, calculate Pearson's Correlation coefficient (r) for the relationship between the time in months (t) and the number of long-footed potoroos (N).

(1 mark)

(ii) Hence, state why it would be considered appropriate to make predictions based on a linear model for this data.

(1 mark)

(b) Tick the model which best represents the equation of the least squares regression line (line of best fit, $y = ax + b$) for the linear model, using the data from Table 2.

☐ $N = 2.01t + 79.5$ ☐ $N = 1.02t + 85.4$ ☐ $N = 79.5t + 2.01$ (1 mark)

(c) Using your model from part (b),

(i) complete the following statement to describe the a value of your least squares regression line.

For every additional month in the breeding program, the number of potoroos

will ________________ by ________________.

(1 mark)

(ii) predict the time when the number of potoroos would reach 300 in the breeding program.

(1 mark)

At the end of the 5-year breeding program, the potoroos were released into their natural habitat.

Table 3 below shows the number of potoroos over the next 4.5 years in their natural habitat.

Table 3: Number of long-footed potoroos in their natural habitat (N)

Months after release (M)	0	6	12	18	24	30	36	42	48	54
Number of long-footed potoroos (N)	199	208	225	204	261	276	294	312	328	341

(d) A recording error has occurred in the collection of this data and an outlier is present.

Which of the following is the outlier? Tick the appropriate box to indicate your answer.

☐ (0, 199)

☐ (18, 204)

☐ (48, 328) (1 mark)

(e) Remove the outlier that you have selected in part (d).

Using correct variables, state the least squares regression equation (line of best fit, $y = ax + b$) for the remaining data in Table 3.

(2 marks)

(f) Using your model from part (e), predict the time that it will take for the number of potoroos to reach 300 after being released into their natural habitat.

(1 mark)

(g) Using your answers from parts (c)(ii) and (f), comment on the length of time it takes for the number of potoroos to reach 300 if they were left in the breeding program rather than being released into their natural habitat at the end of the breeding program.

(2 marks)

Question 5 (12 marks)

Table 4 below shows the number of hours of training that four apprentice electricians have been given in completing various tasks as part of their apprenticeship. The apprentices work for the same company.

Table 4: Hours of training given to apprentice electricians

	Diagnosing wiring issues	*Installing systems*	*Repairing systems*	*Interpreting drawings*
Benji	57	55	66	61
Jacob	73	65	79	75
Kelly	82	88	95	90
Sarah	38	79	82	38

(a) Which statement below is the correct interpretation of the value 73 in the shaded cell?

Tick the appropriate box to indicate your answer.

- ☐ Jacob spends 73 days diagnosing wiring issues.
- ☐ Jacob is 73 years old.
- ☐ Jacob has had 73 hours of training in diagnosing wiring issues.
- ☐ Jacob has 73 drawings to interpret. (1 mark)

(b) Calculate the total number of hours of training that Benji has had as an apprentice electrician, based on the information given in Table 4 above.

(1 mark)

The four apprentices will be working on a major company project where the four tasks need to be completed at the same time.

(c) For each task, the project manager wishes to allocate the apprentice with the most hours of training.

State why this is **not** possible.

(1 mark)

The Hungarian algorithm will be applied to maximise the total combined hours of training that the four apprentice electricians have been given.

The original array below has been reduced in order to perform the Hungarian algorithm.

Original array

57	55	66	61
73	65	79	75
82	88	95	90
38	79	82	38

→

Reduced array

38	40	29	34
22	30	16	20
13	7	0	5
57	16	13	57

(d) (i) State the process that was used to go from the original array to the reduced array above.

(1 mark)

(ii) Complete the remaining steps of the Hungarian algorithm below. Show the result of each step clearly.

38	40	29	34
22	30	16	20
13	7	0	5
57	16	13	57

(4 marks)

(iii) Hence complete Table 5 below, stating the name of the apprentice allocated to each task for the two optimal solutions.

Table 5

	Diagnosing wiring issues	*Installing systems*	*Repairing systems*	*Interpreting drawings*	*Total experience (hours)*
Solution 1					
Solution 2					

(3 marks)

(e) State *one* limitation of using the Hungarian algorithm to allocate tasks to the apprentices in this context.

(1 mark)

Question 6 (18 marks)

Grace took out a home loan of $\$356\,000$ for 25 years. The loan had an interest rate of 5.11% per annum, compounded monthly.

(a) (i) Show that Grace would pay approximately $\$2100$ per month in loan repayments.

(2 marks)

(ii) Calculate the total interest that Grace would be expected to pay for the 25-year loan.

(1 mark)

(iii) State *two* reasons why your solution to part (a)(ii) could be considered unreasonable.

(2 marks)

(b) Show that the remaining balance on the loan after 3 years is approximately $\$333\,000$.

(1 mark)

At this time (3 years after the start of the loan), Grace won $\$50\,000$ in a lottery. She has the option to immediately deposit the $\$50\,000$ in an offset account linked to her home loan where it will stay for exactly 4 years.

(c) (i) State the new balance that interest will be calculated on as soon as the $\$50\,000$ is deposited into the offset account.

(1 mark)

(ii) Calculate the remaining balance after 4 years, assuming that repayments do not change and that the full $\$50\,000$ remains in the offset account.

(2 marks)

(iii) Calculate the interest saved over the duration of the loan by placing the $\$50\,000$ in the offset account for 4 years.

(3 marks)

A friend suggested that Grace should instead invest the $\$50\,000$ in an investment account with a return of 4.79% per annum, compounded fortnightly.

(d) (i) Show that the balance of the investment after 4 years is approximately $\$60\,500$.

(2 marks)

(ii) Hence, calculate the interest earned on the investment over the 4 years.

(1 mark)

(iii) Grace has a marginal tax rate of 37%.

Calculate the total tax to be paid on the 4-year investment. Assume that the tax is paid once at the end of the investment.

(1 mark)

(e) With reference to your solutions from parts (c) and (d), which option would be better for Grace? Tick the appropriate box to indicate your answer, and provide *two* financial reasons to support your answer.

☐ Offset account ☐ Investment account

(2 marks)

Question 7 (10 marks)

Magicians spend time learning and practising new tricks. Table 6 below shows the time spent studying (in hours) and the number of new tricks mastered by Sensational Sam.

Table 6: Time spent by Sensational Sam studying and mastering new tricks

Hours studied (H)	10	20	40	60	80	100
Tricks mastered (T)	4	7	10	19	27	48

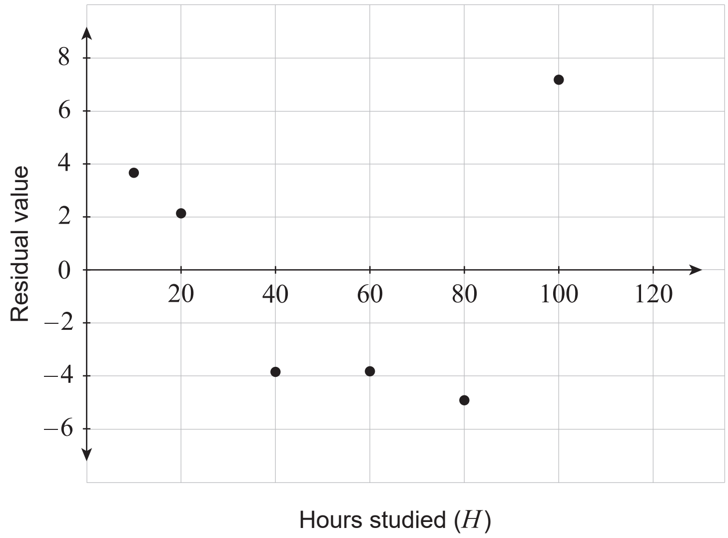
The linear regression equation for Sensational Sam's data is $T = 0.45H - 4.09$.

(a) Complete Table 7 below by adding the missing residual value for the linear model.

Table 7

Hours studied (H)	*Residual value*
10	3.59
20	2.09
40	–3.92
60	–3.92
80	
100	7.07

Residual plot for linear model



(1 mark)

(b) (i) Complete Table 8 below for Sensational Sam's data.

Table 8

Model	*Regression equation*	r^2
Linear ($y = ax + b$)	$T = 0.45H - 4.09$	
Exponential ($y = a.b^x$)	$T = 3.58 \times (1.026)^H$	

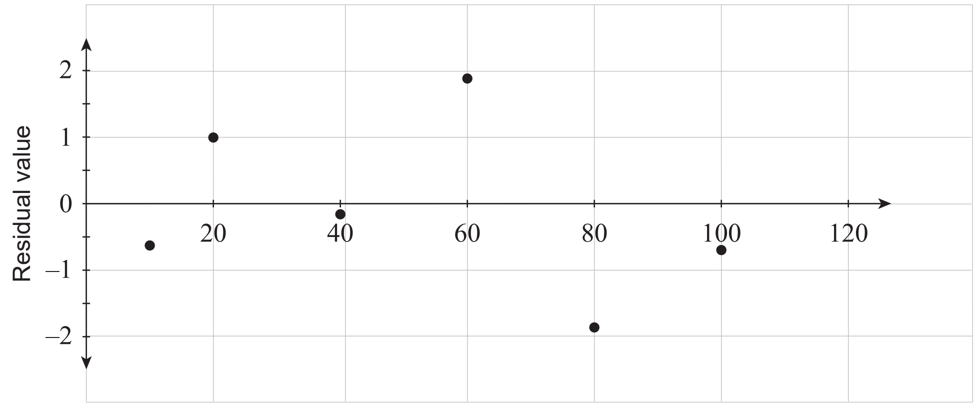
(2 marks)

(ii) Using evidence from parts (a) and (b)(i), discuss which model best fits Sensational Sam's data.

(2 marks)

The residual plot for the exponential model is shown below.

Residual plot for exponential model



(c) *Circle* the corresponding residual point on the graph above that results in the greatest underestimate of tricks mastered, based on the exponential model for this data. (1 mark)

Marvellous Max has been training for a similar time to Sensational Sam and the tricks mastered by Max can be given by the exponential model $T = 2.14 \times (1.052)^H$.

(d) Interpret the values of a and b in the context of this problem.

(2 marks)

(e) Predict the number of hours of study required for Marvellous Max to master 48 tricks.

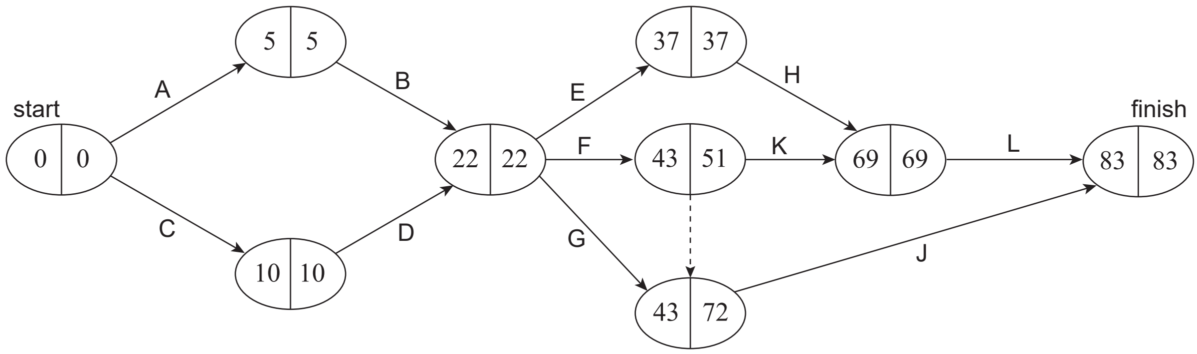
(1 mark)

(f) Using graphs of the two exponential models provided (or by some other method), predict after how many hours of study Sensational Sam and Marvellous Max will have mastered the *same* number of tricks.

(1 mark)

Question 8 (9 marks)

The network diagram below shows the tasks to be completed so that students on a camp can start their hike. The times taken (t) for completing tasks are in minutes, where $t > 0$ minutes.



(a) State the time taken to complete task C.

(1 mark)

(b) Tick the correct option to complete the statement below.

The total time taken to complete tasks A and B is

☐ shorter than the total time taken to complete tasks C and D.

☐ longer than the total time taken to complete tasks C and D.

☐ the same as the total time taken to complete tasks C and D. (1 mark)

(c) State the time taken to complete task K.

(1 mark)

(d) (i) State the latest starting time for task F.

(1 mark)

(ii) Which of the following tasks has the greatest slack time? Tick the appropriate box to indicate your answer.

☐ Task D

☐ Task F

☐ Task K

☐ Task J (1 mark)

(e) The time taken to complete task G cannot be calculated from the network diagram.

Complete the following statement.

Task G will take more than ________ minutes, but at most ________ minutes, to complete.

(1 mark)

(f) (i) There are two critical paths within this network.

State **all** tasks that are critical within the network.

(1 mark)

(ii) What effect will reducing task D by 3 minutes have on the critical path(s) and overall completion time for this network?

(2 marks)

You may write on this page if you need more space to finish your answers. Make sure to label each answer carefully (e.g. 8(f)(ii) continued).