## Test: Motion

Projectile Motion, Forces and Momentum, Circular Motion and Gravitation
1.
(a) (arrow going directly downwards)
(b) $\quad v_{\mathrm{H}}=5.0 \mathrm{~ms}^{-1}$

(c) $\theta=\tan ^{-1}\left(\frac{v_{V}}{v_{H}}\right)=\tan ^{-1}\left(\frac{6.0}{5.0}\right)=50^{\circ}$ below horizontal
$v=\sqrt{v_{H}{ }^{2}+v_{V}{ }^{2}}=\sqrt{5.0^{2}+6.0^{2}}=7.8 \mathrm{~ms}^{-1}$
(d) Air resistance opposes motion, so it slows both the horizontal and vertical components of velocity. The slowing of the vertical component of velocity means the projectile takes longer to fall to the ground, time of flight is increased.
(e) (arrow going directly downwards)
(f)
(i) No effect
(ii) Greater
2.
(a) $v_{0_{H}}=v_{0} \cos \theta=21 \cos 34^{\circ}=17 \mathrm{~ms}^{-1}$

$$
v_{0_{V}}=v_{0} \sin \theta=21 \sin 34^{\circ}=12 \mathrm{~ms}^{-1}
$$

(b) $s=v_{0} t+\frac{1}{2} a t^{2}$

Using the vertical component, $s_{V}=0$ (launched from ground)

$$
\begin{aligned}
& \therefore 0=v_{0} t+\frac{1}{2} a t^{2} \\
& \therefore 0=t\left(v_{0}+\frac{1}{2} a t\right) \\
& \therefore t=0 \text { or } v_{0}+\frac{1}{2} a t=0
\end{aligned}
$$

We want time of flight so ignore $t=0$
$\therefore t=\frac{-v_{0}}{\frac{1}{2} a}=\frac{-12}{\frac{1}{2} \times-9.8}=2.4 \mathrm{~s}$
(c) Using the horizontal component, $a_{H}=0$
$\therefore s=v_{0} t=17 \times 2.4=42 \mathrm{~m}$
(d) $v^{2}=v_{0}{ }^{2}+2 a s$

Using the vertical component, $v=0$ at max height
$0=v_{0}{ }^{2}+2 a s$
$\therefore s=\frac{-v_{0}{ }^{2}}{2 a}=\frac{-12^{2}}{2 \times-9.8}=7.0 \mathrm{~m}$
3.
(a) $p=m v=0.0310 \times 4.20=0.1302 \mathrm{kgms}^{-1}$


$$
\Delta \vec{p}=\vec{p}_{f}-\vec{p}_{i}
$$

Triangle is equilateral so $\Delta p=0.130 \mathrm{kgms}^{-1}$ to the left
(b) $\vec{F}=\frac{\Delta \vec{p}}{\Delta t}=\frac{0.130}{2.00 \times 10^{-2}}=6.50 \mathrm{~N}$ to the left
4.

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$\otimes \quad \otimes p_{\mathrm{Bf}} \otimes \otimes$



The magnitude and direction of the initial and final total momenta are equal, so momentum has been conserved.
5.
(a) $0 \mathrm{kgms}^{-1}$
(b) Momentum conserved $\left(\vec{p}_{f}=\vec{p}_{i}\right)$ so $\vec{p}_{A}+\vec{p}_{B}+\vec{p}_{C}=0$

(c) $1.0 \times v=\sqrt{8.0^{2}-4.0^{2}}$
$\therefore v=6.9 \mathrm{~ms}^{-1}$
6. Each expelled ion is given momentum - but the according to the law of conservation of momentum the total spacecraft-ions system cannot have changed its momentum, so the ship is given momentum in the opposite direction to cancel it out. Since this is happening at a constant rate, the ship has constant acceleration.
7.
(a) $v=\frac{2 \pi r}{T}=\frac{2 \pi \times 0.12}{1.2}=0.63 \mathrm{~ms}^{-1}$
(b) $F=m a=m \frac{v^{2}}{r}=0.016 \times \frac{0.63^{2}}{0.12}=0.053 \mathrm{~N}$
8.
(a) (arrow pointing up to the left, perpendicular to the road)
(b) The banking angle causes a non-vertical normal force on the car (as shown above) which has a horizontal component directed towards the centre of the curve. The horizontal component acts therefore supplies some centripetal force, reducing the amount that would usually be required by friction.
9.
(a) $F_{1}=3.9 \times 10^{22} \mathrm{~N}$

Let $M_{1}=M$
$\therefore M_{2}=2 M$
$F=G \frac{M m}{r^{2}}$
G, $m, r$ constant
$\therefore F \propto M$
$\therefore \frac{F_{2}}{F_{1}}=\frac{M_{2}}{M_{1}}$
$\therefore F_{2}=F_{1} \times \frac{M_{2}}{M_{1}}=3.9 \times 10^{22} \times \frac{2 \mathrm{M}}{M}$
$=3.9 \times 10^{22} \times 2=7.8 \times 10^{22} \mathrm{~N}$
(b) As required by Kepler's First Law, this orbit has an elliptical path with Star 2 at one focus. The other focus (somewhere between Star 1 and Star 2 in this diagram) is empty.
(c) Star 1's speed will be greater at point B because it is closer to Star 2 than at point A and therefore will need to travel a greater distance in order for the line from Star 1 to Star 2 to sweep out an equal area. Kepler's Second Law states that these equal areas are swept in equal time interval.
10.
(a) Let the satellite's mass be $m$, and the planet's mass $M$. .
$F=G \frac{m M}{r^{2}} \quad$ and $\quad F=m a=m \frac{v^{2}}{r}$
A satellite's centripetal force is provided by gravitation.
$\therefore G \frac{m M}{r^{2}}=m \frac{v^{2}}{r}$
$\therefore G \frac{M}{r}=v^{2}$
$\therefore v=\sqrt{\frac{G M}{r}}$
(b) Let Earth's mass be $M$.
$v=\sqrt{\frac{G M}{r}}$
$\therefore M=\frac{v^{2} r}{G}$
Inserting the data for Satellite B:
$M=\frac{3072^{2} \times 4.224 \times 10^{7}}{6.67 \times 10^{-11}}=5.98 \times 10^{24} \mathrm{~kg}$
(c) $v=\sqrt{\frac{G M}{r}}=\sqrt{\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{2.112 \times 10^{7}}}=4.35 \times 10^{3} \mathrm{~ms}^{-1}$
(d) $a=\frac{v^{2}}{r}=\frac{(3072)^{2}}{4.224 \times 10^{7}}=0.2234 \mathrm{~ms}^{-2}$
11.
(a) $\sqrt{\frac{G M}{r}}=\frac{2 \pi r}{T}$
$\therefore \frac{G M}{r}=\frac{2^{2} \pi^{2} r^{2}}{T^{2}}$
$\therefore r^{3}=\frac{G M T^{2}}{4 \pi^{2}}$
$\therefore r=\sqrt[3]{\frac{G M T^{2}}{4 \pi^{2}}}$
(b) Geostationary orbits must be at a specific altitude/radius, polar orbits may have any radius. Geostationary orbits must have one particular speed/period, polar orbits can have any Geostationary orbits are around the equator, polar orbits are from pole to pole.
(c) The force acting on a satellite is gravitation, which acts between the centres of mass of objects, meaning the satellite is pulled towards the centre of the Earth. A centripetal force is towards the centre of the circle of motion, and for a satellite the centripetal force is provided by the force of gravitation, therefore their directions must coincide.
12.

Communication and Collaboration: The TESS project required collaboration between scientists and organisations, including NASA, MIT, STSI and Orbital ATK. The launch itself was performed by SpaceX, another independent company.
Development: The TESS project includes new technologies for data collection which have previously not been available, such as wide-field cameras and detectors. New evidence from these instruments may lead to new discoveries or understandings.
Application and Limitation: The TESS project so far has demonstrated how the use of scientific knowledge can lead to unexpected results. Such results have included, for example, types of planets which were not predicted to exist.

