Trigonometry: Right-Angled Triangles

Prior Knowledge:

- Using Pythagoras' theorem.
- Substitution into a formula.
- Rearranging a formula.
- Rounding to decimal places and significant figures.
- Angles on a straight line and at a point.

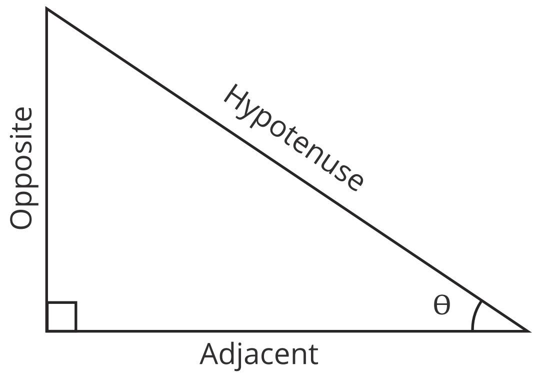
Right-angled trigonometry allows you to find a missing angle or side in a right-angled triangle when given two sides, or an angle and a side. Trigonometry uses the three trigonometric functions: sine, cosine and tangent, which you will see written on your calculator as **sin**, **cos** and **tan**. Each function can be expressed as a ratio of the length of two sides:

$$\sin(\theta) = \frac{O}{H} \qquad \cos(\theta) = \frac{A}{H} \qquad \tan(\theta) = \frac{O}{A}$$

Where θ is the measure of the angle, **O** is the length of the opposite side, **A** is the length of the adjacent side and **H** is the length of hypotenuse.

Of these three sides, the hypotenuse is always the side opposite the right angle and is always the longest side (this can be a good tool to check your answer makes sense).

Opposite and adjacent sides are not fixed; they depend on the angle you're interested in. The opposite side is farthest from the angle (but not the hypotenuse). The adjacent side is next to the angle (but, again, not the hypotenuse).



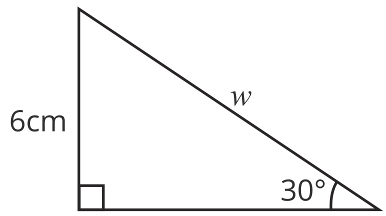
You need to be able to remember the three formulae above; they won't be given to you in an exam. A common way of doing this is to remember them as a word: **SOHCAHTOA**. This can be written out to help rearrange the formulae:

$$\frac{\mathbf{O}}{\mathbf{S} \times \mathbf{H}} \qquad \frac{\mathbf{A}}{\mathbf{C} \times \mathbf{H}} \qquad \frac{\mathbf{O}}{\mathbf{T} \times \mathbf{A}}$$

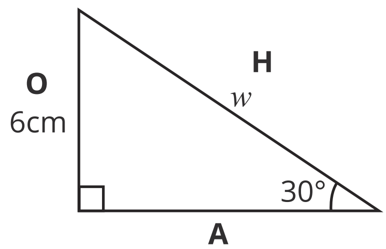
Finding a Missing Side

Your first step in finding the length of a missing side will be to label the sides:

Example 1: Find the length of the missing side, w.



In this case, side w is the hypotenuse (**H**) because it is farthest from the right angle. The side labelled 6cm is the opposite (**O**) side because it is farthest from the angle we will use (labelled 30°). The side without a label or a length is the adjacent (**A**) because it is next to the angle we will use.



The second step will be to choose the relevant formula. In most examples, you will be given two values and you will be asked to find a third value. In this case, we have been given the angle and the opposite (**O**) and we are interested in the length of the hypotenuse (**H**). We do not need the adjacent side to answer this question so we need to use the trigonometric formula that includes **O** and **H**:

$\sin(\theta) = \frac{O}{H}$

We are interested in the length of the hypotenuse (**H**) so we need to rearrange our formula:

$H = \frac{O}{\sin(\theta)}$

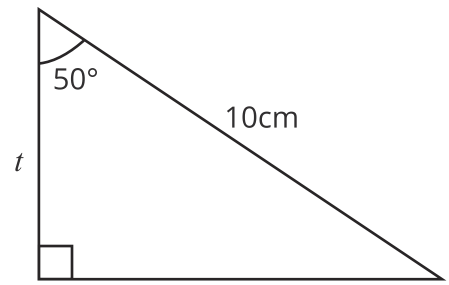
> **Hint:**
>
> You can also do this step by using the formula triangle: $\frac{O}{S \times H}$
>
> Cover **H** – this tells you that you need to calculate $\frac{O}{S}$.

Finally, substitute the values and calculate the answer (if you choose, you can substitute the values and then rearrange – this will not affect the answer):

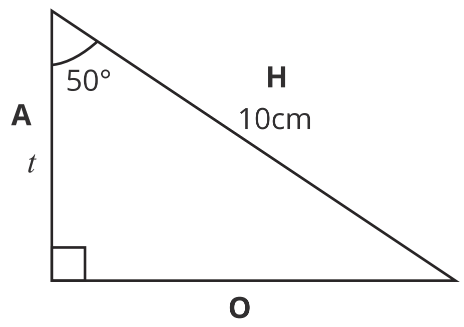
$H = \frac{6}{\sin(30°)}$

$H = 12\text{cm}$

Example 2: Find the length of side t. Give your answer correct to 3s.f.



Always start by labelling the sides:



In this case, we know the length of the hypotenuse and we want to know the length of the adjacent, so we use cosine:

$\cos(\theta) = \frac{A}{H}$

We rearrange the formula to find A:

$A = \cos(\theta) \times H$

Substitute the values:

$t = \cos(50°) \times 10$

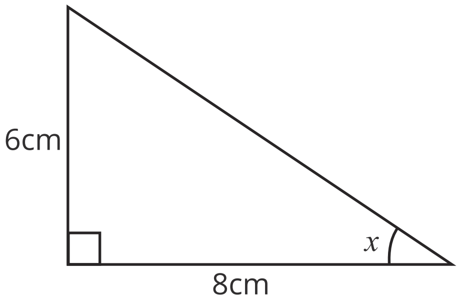
Calculate the answer:

$t = 6.43$cm (to 3s.f.)

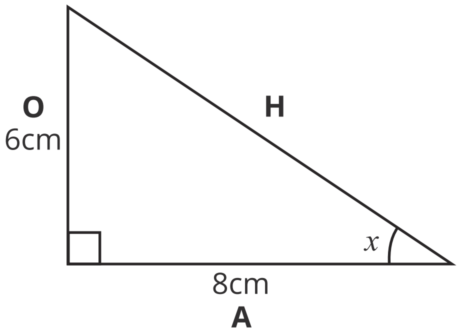
Finding a Missing Angle

In a similar way, you can find a missing angle when given two sides:

Example 3: Find the size of angle x. Give your answer correct to 3s.f.



As before, start by labelling the sides:



In this case, we are interested in the opposite and the adjacent. Choose the correct function:

$\tan(\theta) = \frac{O}{A}$

We don't need to rearrange the formula because tan(θ) will give us the angle we are interested in. We just need to substitute our values:

$\tan(x) = \frac{6}{8}$

Unlike finding a side, we still have one more step. We are interested in the size of the angle x but we still only know $\tan(x)$. To find x, we need to carry out an inverse tangent function, written as $\tan^{-1}$:

$x = \tan^{-1}(\frac{6}{8})$ ($\tan^{-1}(0.75)$ or $\tan^{-1}(\frac{3}{4})$ will also work)

$x = 36.8698...$

$x = 36.9°$ (to 3s.f.)

If you were using the sine function to find x, you would use $\sin^{-1}$. If you were using the cosine function, you would use $\cos^{-1}$.

Your turn:

1. In each question, use the correct inverse trigonometric function to find the value of the angle, θ, correct to 1d.p.

a. $\sin(\theta) = 0.4$

c. $\tan(\theta) = 2.87$

e. $\cos(\theta) = 0$

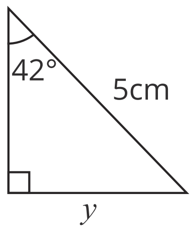
b. $\cos(\theta) = -0.78$

d. $\sin(\theta) = 1$

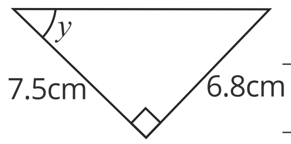
f. $\tan(\theta) = 0.68$

2. In each question, find the value of y. Give your answers correct to 3s.f.

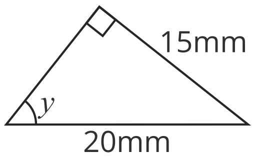
a.



e.



b.



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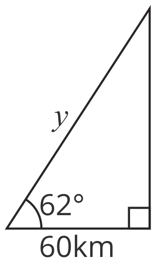
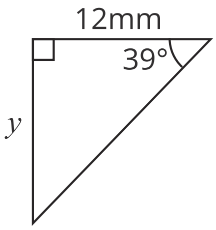
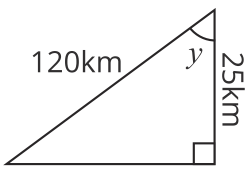


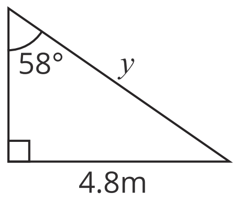
c.



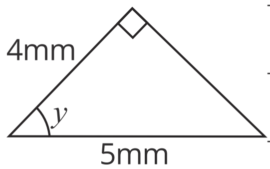
g.



d.



h.



3. A window cleaner needs to clean a window. His ladder is 4m long. For safety reasons, his ladder needs to make a 75° angle with the ground (which is horizontal). How far from the wall should he place the base of his ladder? Give your answer correct to 1d.p.

4. A plane takes off from an airfield, **A**, flies 300km due north to point **B**, before turning to fly directly east for 800km to point **C**. It then turns to the right and flies in a straight line back to the airfield. How many degrees does the plane turn through at point **C**? Give your answer correct to 1d.p.

BEYOND MATHS

5. Find the size of angle x. Give your answer correct to 2s.f.