Using Proportionality

Proportionality can be used to solve any problem where two quantities are proportional (all other variables are constant).

For example:

In uniform circular motion according to $v = \frac{2\pi r}{T}$, v is proportional to $\frac{1}{T}$ since 2, π and r are all constant.

So $v \propto \frac{1}{T}$. When two variables are proportional, their ratio is constant, so

$$\frac{v}{\left(\frac{1}{T}\right)} = k \qquad \text{(where } k \text{ is a constant)}$$

Another way of thinking about this relationship (simply a rearrangement of the above) is that

$$v = k\frac{1}{T} \qquad \text{(see how } k \text{ represents all constants in the original formula)}$$

In general:

If $a \propto b$, then $\frac{a}{b} = k$ where k is a constant.

Knowing that the ratio is constant allows for problem solving.

Example 1: Cars A and B are driving around a curve. Their speeds are $1.5v$ and v respectively. Using proportionality, calculate the ratio $T_A : T_B$ of the time it takes each car to drive the curve.

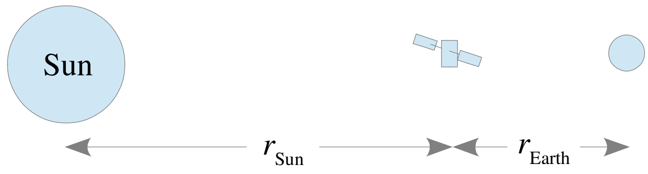
$v \propto \frac{1}{T} \qquad \therefore \frac{v}{\left(\frac{1}{T}\right)}$ is constant.

$$\therefore \frac{v_A}{\left(\frac{1}{T_A}\right)} = \frac{v_B}{\left(\frac{1}{T_B}\right)} \qquad \therefore \frac{1.5v}{\left(\frac{1}{T_A}\right)} = \frac{v}{\left(\frac{1}{T_B}\right)} \qquad \therefore 1.5T_A = T_B$$

$$\therefore \frac{T_A}{T_B} = \frac{1}{1.5}$$

$$\therefore T_A : T_B = 1:1.5$$

Example 2: A satellite can be placed between the Sun and the Earth such that the net gravitational force on the satellite is zero. Given that M_{Sun} is equal to $332900 \times M_{\text{Earth}}$, use proportionality to calculate the ratio $\frac{r_{\text{Sun}}}{r_{\text{Earth}}}$ of the distances from the satellite.



Let the mass of the satellite be m

$$\therefore F = G\frac{mM}{r^2} \quad \therefore r = \sqrt{G\frac{mM}{F}}$$

The net force on the satellite is zero, so F is equal in both directions, so we can treat it as constant.

m and G are also constant, $\therefore r \propto \sqrt{M}$

$\therefore \frac{r}{\sqrt{M}}$ is constant (proportional means their ratio is constant)

$$\therefore \frac{r_{\text{Earth}}}{\sqrt{M_{\text{Earth}}}} = \frac{r_{\text{Sun}}}{\sqrt{M_{\text{Sun}}}}$$

$$\therefore \frac{r_{\text{Sun}}}{r_{\text{Earth}}} = \frac{\sqrt{M_{\text{Sun}}}}{\sqrt{M_{\text{Earth}}}} = \frac{\sqrt{332900 M_{\text{Earth}}}}{\sqrt{M_{\text{Earth}}}} = \sqrt{332900} = 577$$