# Proportionality Worksheet 1

*Direct Proportionality*

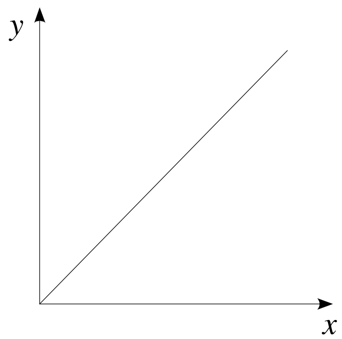
**Learning Intention:** To be able to write a *direct* proportionality relationship and slope of the graph, from a formula.

In Physics investigations, the relationship between an *independent* variable and a *dependent* variable is determined by changing the independent and measuring the dependent.

A formula or equation often has more than two variables, and we only want to measure the effect of one variable on another – so we must keep everything else constant.

If the two variables that are changing are *directly proportional*, then changing one causes a consistent change in the other. This is true if the equation can be written in the form $y = kx$ where $k$ is constant.
The phrase *"$y$ is directly proportional to $x$"* can be written $y \propto x$
(The equals changes to a proportionality symbol "$\propto$" and the constants are removed from the equation).

If two variables are directly proportional, the graph will be a straight line through the origin:




The slope of the graph is equal to $k$

**Important note:** Not every straight line graph is a direct proportionality. If the graph doesn't pass through the origin (0,0) then the relationship is called linear dependent and the equation is of the form $y = mx + c$

Direct proportionality examples:

1. If $a = 3b$, then $a \propto b$ because 3 is constant. The slope of the graph will be 3.
2. If $s = \frac{1}{3}t$, then $s \propto t$ because $\frac{1}{3}$ is constant. The slope of the graph will be $\frac{1}{3}$.
3. If $g = Rh$ and $R$ is held constant, then $g \propto h$. The slope of the graph will be $R$.
4. If $w = \frac{2nQ}{5f}$ and both $n$ and $f$ are held constant, then $w \propto Q$ because 2 and 5 are also constant.

   The slope of the graph will be $\frac{2n}{5f}$.

Practice:

Write the proportionality relationship and the slope of the graph for each of the following:

1. $n = 5m$
2. $A = \frac{2}{5}B$
3. $T = 3Lb$, if $b$ is held constant.
4. $d = \frac{6abc}{e}$, if $b$, $c$ and $e$ are all held constant.

# Proportionality Worksheet 2

*Inverse Proportionality*

**Learning Intention:** To be able to write an *inverse* proportionality relationship and slope of the *transformed* graph, from a formula.

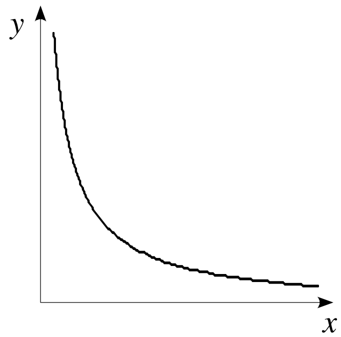
If the two variables that are changing are *inversely proportional*, then changing one causes the opposite change in the other, for example decreasing one increases the other.

This is true if the equation can be written in the form $y = k\frac{1}{x}$ or $y = \frac{k}{x}$ where $k$ is constant.

The phrase *"y is inversely proportional to x"* can be written $y \propto \frac{1}{x}$

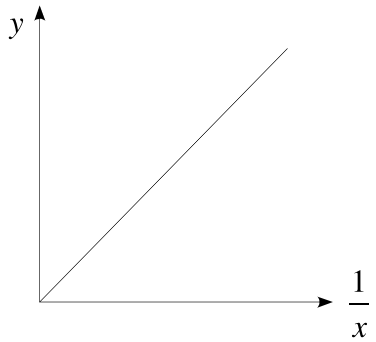
(The equals changes to a proportionality symbol "$\propto$" and the constants are removed from the equation).

If two variables are inversely proportional, the graph of the two variables will have this shape:




We can't find the equation of this graph, so we need to *transform* the data into a straight line form first.

*y* is *directly* proportional to $\frac{1}{x}$ so we graph these:




The slope of the graph is equal to $k$

Inverse proportionality examples:

1. If $a = \frac{3}{b}$, then $a \propto \frac{1}{b}$ because 3 is constant. The slope of the transformed graph will be 3.
2. If $w = \frac{2nQ}{5f}$ and both $n$ and $Q$ are held constant, then $w \propto \frac{1}{f}$ because 2 and 5 are also constant.

   The slope of the the transformed graph will be $\frac{2nQ}{5}$

Practice:

Write the proportionality and the slope of the transformed graph for each of the following:

1. $n = \frac{5}{m}$

2. $d = \frac{6abc}{e}$ , if $a$, $b$, and $c$ are all held constant.

# Proportionality Worksheet 3

*Other Proportionalities*

**Learning Intention:** To be able to *predict* a proportionality relationship and find an equation, from *data*.

The rules we have seen so far can be applied to many relationships. Here are some more examples of equations and their proportionality. In each case $k$ is held constant.

| **Equation** | **Proportionality** |
|---|---|
| $y = kx^2$ | $y \propto x^2$ "$y$ is proportional to the square of $x$" |
| $y = \frac{k}{x^2}$ | $y \propto \frac{1}{x^2}$ "$y$ is proportional to the inverse square of $x$" |
| $y = kx^3$ | $y \propto x^3$ "$y$ is proportional to the cube of $x$" |
| $y = k\sqrt{x}$ | $y \propto \sqrt{x}$ "$y$ is proportional to the square root of $x$" |
| $y = \frac{k}{\sqrt{x}}$ | $y \propto \frac{1}{\sqrt{x}}$ "$y$ is proportional to the inverse square root of $x$" |

For each of the following tables of data:
1. Plot a graph of *dependent* variable against *independent* variable to determine the likely relationship.
2. If the relationship is not directly proportional:
   (a) Fill out a column with transformed values if necessary.
   (b) Graph the transformed values.
3. Find the slope of the line of best fit (and hence the constant of proportionality).
4. Find the equation for the original data.

*Cross out the middle column if you don't need it. Also, don't worry about units for this exercise.*

**Table 1**

| $a$ | | $b$ |
|---|---|---|
| 1.0 | | 3.1 |
| 2.0 | | 6.3 |
| 3.0 | | 9.4 |
| 4.0 | | 12.6 |
| 5.0 | | 15.7 |

Likely relationship:

Slope:

Equation:

**Table 2**

| $m$ | | $n$ |
|---|---|---|
| 1.0 | | 0.50 |
| 2.0 | | 0.26 |
| 3.0 | | 0.17 |
| 4.0 | | 0.14 |
| 5.0 | | 0.12 |

Likely relationship:

Slope:

Equation:

**Table 3**

| $i$ | | $j$ |
|---|---|---|
| 1.0 | | 2.4 |
| 2.0 | | 8.2 |
| 3.0 | | 18.7 |
| 4.0 | | 32.8 |
| 5.0 | | 50.3 |

Likely relationship:

Slope:

Equation:

# Proportionality Problem Solving

**Learning Intention:** To be able to use proportionality to *solve problems* involving ratios.

If everything else except the independent and dependent variables are kept constant, then the ratio between the variables is constant (the fraction stays the same).

For example, if $y = kx$, where $k$ is constant, then $k = \frac{y}{x}$ for all pairs of values for $x$ and $y$, because $y \propto x$.

So $\frac{y_1}{x_1} = k$ , $\frac{y_2}{x_2} = k$ , etc., which means that $\frac{y_1}{x_1} = \frac{y_2}{x_2}$. This rearrangement can be useful for solving problems.

Even if the proportionality is not direct, the same rearrangement can be applied.

For example, if $y = kx^2$, where $k$ is constant, then $k = \frac{y}{x^2}$ for all pairs of values for $x$ and $y$, because $y \propto x^2$.

So $\frac{y_1}{x_1^2} = k$ , $\frac{y_2}{x_2^2} = k$ , etc., which means that $\frac{y_1}{x_1^2} = \frac{y_2}{x_2^2}$.

Proportionality problem solving examples:

1. Consider the formula $F = Eq$, with $E$ held constant.
   If $F = 4$ for some value of $q$, calculate $F$ if $q$ is doubled.

$$F_1 = 4 \quad \text{for} \quad q_1$$
$$F_2 = ? \quad \text{for} \quad q_2 = 2q_1$$
$$F \propto q$$
$$\therefore \frac{F_1}{q_1} = \frac{F_2}{q_2}$$
$$\therefore F_2 = \frac{q_2 F_1}{q_1} = \frac{2q_1 \times 4}{q_1} = 8 \quad \{\, q_1 \text{ cancels} \,\}$$

2. Consider the formula $a = \frac{v^2}{r}$, with $r$ held constant.
   If $a = 24$ for some value of $v$, calculate $a$ if $v$ is halved.

$$a_1 = 24 \quad \text{for} \quad v_1$$
$$a_2 = ? \quad \text{for} \quad v_2 = \tfrac{1}{2} v_1$$
$$a \propto v^2$$
$$\therefore \frac{a_1}{v_1^2} = \frac{a_2}{v_2^2}$$
$$\therefore a_2 = \frac{a_1 v_2^2}{v_1^2} = \frac{24 \times \left(\tfrac{1}{2} v_1\right)^2}{v_1^2} = \frac{24 \times \tfrac{1}{4} v_1^2}{v_1^2} = 6 \quad \{\, v_1^2 \text{ cancels} \,\}$$

Practice:

1. Consider the formula $W = Fd$, with $d$ held constant.
   If $W = 120$ for some value of $F$, calculate $W$ if $F$ is halved.
2. Consider the formula $K = \frac{1}{2} m v^2$, with $m$ held constant.
   If $K = 4$ for some value of $v$, calculate $K$ if $v$ is doubled.
3. Consider the formula $F = ma$, with $F$ held constant.
   If $a = 8$ for some value of $m$, calculate $a$ if $m$ is doubled.