Section 1: Motion in Two Dimensions

Two types of motion in two dimensions are studied: projectile motion and uniform circular motion. These topics prepare students for an understanding of the vector form of Newton's second law of motion. In projectile motion, in the absence of air resistance, acceleration is constant in magnitude and direction, whereas in uniform circular motion acceleration is constant in magnitude but not in direction. In each case, acceleration is in the direction of the net force causing it.

Newton's law of universal gravitation is introduced, and used to describe the motion of satellites in circular orbits.

The directional relationship between acceleration and net force leads to a vector statement of Newton's second law of motion for situations involving motion in more than one dimension. The law is expressed in terms of momentum and is used in conjunction with Newton's third law to obtain a vector expression for the law of conservation of momentum for two interacting particles. The fact that the law of conservation of momentum is a fundamental conservation law that applies to any number of particles is presented without proof.

Topic 1: Projectile Motion

In the absence of air resistance, and moving under the action of a constant gravitational force, a projectile has a constant acceleration in the direction of the force. The horizontal component of velocity of such a projectile is constant, and the vertical component changes at a constant rate. The time of flight and the range of the projectile are calculated, and the effect of air resistance on the motion is treated qualitatively. These key ideas are applied to projectiles in sport (e.g. a shot put).

Key Ideas	Intended Student Learning
Students should know and understand the following:	*Students should be able to do the following:*
Vertical and Horizontal Components of Velocity	
For a projectile, in the absence of air resistance, the: • horizontal component of velocity is constant • acceleration is in the vertical direction and is the same as that of a vertically free-falling object.	Given a multi-image photograph of a projectile, demonstrate that the: • horizontal component of velocity is constant • acceleration is in the vertical direction and is the same as that of a vertically free-falling object.
The horizontal motion and the vertical motion are independent of each other: the constant vertical acceleration is independent of the horizontal speed.	Draw a vector diagram in which the horizontal and vertical components of velocity are added, giving the resultant velocity vector at any instant.
The acceleration of a projectile, in the absence of air resistance, is in the direction of the gravitational force.	Using trigonometric calculations or a scale diagram, calculate, from its horizontal and vertical components, the magnitude and direction of a velocity vector at any instant. On a diagram showing the path of a projectile, draw vectors to represent the velocity and acceleration of the projectile at any instant.

Key Ideas	Intended Student Learning
Students should know and understand the following:	*Students should be able to do the following:*
Determination of the Vertical Component of Velocity	
The equations for constant acceleration in one dimension can be used to calculate the vertical component of velocity of a projectile at any instant.	Given the initial velocity of a projectile, calculate the vertical component of velocity at any instant.
Resolution of Velocity into Components	
Velocity can be resolved into its horizontal and vertical components at any instant.	Using trigonometric calculations or a scale diagram, resolve a velocity vector into its horizontal and vertical components.
Time of Flight	
The time of flight of a projectile is determined by the change in vertical component of velocity and the acceleration.	Calculate the time of flight of a projectile in cases where the final height is the same as the initial height.
Range	
The range of a projectile is calculated by multiplying the horizontal component of velocity and the time of flight.	Using the horizontal component of velocity and the time of flight, calculate the range of a projectile. For a projectile launched from ground height, find, by using sample calculations or otherwise, the: • launch angle that results in the maximum range • relation between the launch angles that result in the same range.
Maximum Height	
The maximum height of a projectile can be calculated from the vertical component of the initial velocity and the acceleration *or* the time of flight and the acceleration.	Using the vertical component of the initial velocity and the acceleration, calculate the maximum height of a projectile. Using the time of flight and the acceleration, calculate the maximum height of a projectile.
Effect of Air Resistance	
Air resistance acts in the opposite direction to the velocity of a projectile at any instant.	Describe how air resistance affects both the horizontal component and the vertical component of velocity and hence the time of flight and range of a projectile.
The magnitude of the force of air resistance on an object depends on the object's shape, size, speed, and surface texture and the density of the air.	Compare qualitatively the force of air resistance acting on different objects.

Key Ideas	Intended Student Learning
Students should know and understand the following:	*Students should be able to do the following:*
Application: Projectiles in Sport	
	Describe and explain the effect of the launch height of a projectile (e.g. a shot put launched from shoulder height) on the maximum range, and the effect of the launch angle for a given height.
	Investigate the extent to which air resistance affects various projectiles in sport.

Topic 2: Uniform Circular Motion

In projectile motion (the example of motion in two dimensions introduced in Topic 1), force and acceleration in the absence of air resistance are constant in both magnitude and direction. This second example of motion in two dimensions involves an object moving with constant speed in a circle (referred to as 'uniform circular motion'). In uniform circular motion the force and acceleration continually change direction and are always directed towards the centre of the circle. The force is always perpendicular to the velocity. The resulting acceleration produces a continual change in the direction of the velocity without changing the magnitude of the velocity. The theory is applied to the banking of road curves.

Key Ideas	Intended Student Learning
Students should know and understand the following:	*Students should be able to do the following:*
Centripetal Acceleration	
The velocity of an object moving with uniform circular motion continually changes direction, and hence the object accelerates. Average acceleration $\vec{a}_{ave}$ for motion in more than one dimension is defined as $\vec{a}_{ave} = \Delta\vec{v}/\Delta t$ where $\Delta\vec{v} = \vec{v}_f - \vec{v}_t$. The acceleration $\vec{a}$ at any instant is obtained by allowing the time interval Δt to become very small. The acceleration of an object moving with uniform circular motion is directed towards the centre of the circle and is called 'centripetal acceleration'. The magnitude of the centripetal acceleration is constant for a given speed and radius and given by $a = v^2/r$.	Using a vector subtraction, show that the change in the velocity $\Delta\vec{v}$, and hence the acceleration, of an object over a very small time interval is directed towards the centre of the circle. Using the relationship $v = 2\pi r/T$, relate the speed v to the period T for a fixed radius. Solve problems involving the use of the equations $a = v^2/r$ and $v = 2\pi r/T$.
Force Causing the Centripetal Acceleration	
A net force directed towards the centre of the circle is necessary to produce the centripetal acceleration.	Describe situations in which the centripetal acceleration is caused by a tension force, a frictional force, a gravitational force, or a normal force.

Key Ideas

Students should know and understand the following:

Application: The Banking of Road Curves

Intended Student Learning

Students should be able to do the following:

Identify the vertical and horizontal forces on a vehicle moving with constant velocity on a flat horizontal road.

Explain that when a vehicle travels round a banked curve at the correct speed for the banking angle, the horizontal component of the normal force on the vehicle (not the frictional force on the tyres) causes the centripetal acceleration.

Derive the equation $\tan\theta = v^2/rg$, relating the banking angle θ to the speed v of the vehicle and the radius of curvature r.

Solve problems involving the use of the equation $\tan\theta = v^2/rg$.

Topic 3: Gravitation and Satellites

The characteristics of gravitational force are examined in this topic. Newton's law of universal gravitation is introduced and used to extend the study of uniform circular motion to the centripetal acceleration caused by the gravitational force on a satellite. These key ideas are applied to weather and communication satellites.

Key Ideas	Intended Student Learning
Students should know and understand the following:	*Students should be able to do the following:*
Newton's Law of Universal Gravitation	
Any two particles experience mutually attractive gravitational forces along the line joining them. The magnitude of these forces is directly proportional to the product of the two masses and inversely proportional to the square of the distance between them. Spherically symmetric objects interact gravitationally as if their masses were located at their centres.	Solve problems involving the use of $F = Gm_1m_2/r^2$, where F is the magnitude of the gravitational forces, m_1 and m_2 are the masses of the particles, r is the distance between them, and G is the constant of universal gravitation. Using proportionality, discuss changes in the magnitude of the gravitational force on each of the masses as a result of a change in one or both of the masses and/or a change in the distance between them. Explain that the gravitational forces are consistent with Newton's third law. Using Newton's law of universal gravitation and second law of motion, calculate the value of the acceleration due to gravity g at a planet or moon.
Satellites in Circular Orbits	
The gravitational force causes the centripetal acceleration when a satellite moves in a circular orbit. For a particular radius of circular orbit there is only one possible speed for a stable satellite orbit.	Demonstrate an understanding that the speed, and hence the period, of a satellite moving in a circular orbit depends only on the radius of the orbit and not on the mass of the satellite. Derive the formula $v = \sqrt{GM/r}$ for the speed v of a satellite moving in a circular orbit of radius r about a spherically symmetric mass M, given that its gravitational effects are the same as if all its mass were located at its centre. Solve problems involving the use of the equations $v = \sqrt{GM/r}$ and $v = 2\pi r/T$.

Key Ideas	Intended Student Learning
Students should know and understand the following:	*Students should be able to do the following:*
Application: Weather and Communication Satellites	
	Explain why the centres of the circular orbits of Earth satellites must coincide with the centre of the Earth.
	Explain why a geostationary satellite must move in a particular orbit of relatively large radius in the Earth's equatorial plane and in the same direction as that in which the Earth rotates.
	Explain the advantages of launching low-altitude equatorial-orbit satellites in a west-to-east direction.
	Explain why low-altitude polar orbits are used in meteorology and surveillance.
	Perform calculations involving orbital periods, radii, altitudes above the surface, and speeds of satellites, including examples which involve the orbits of geostationary satellites.