Topic 4: Momentum in Two Dimensions

A comparison of the directions of the acceleration and of the net force in projectile motion and uniform circular motion suggests the vector form of Newton's second law of motion.

The key implication is that the force on an object determines only the change in its velocity. The new velocity depends on this change, and on the original velocity. The net force on an object therefore changes the speed and/or direction of motion of the object according to a simple vector formula, $\vec{F} = m\vec{a}$.

Newton's second law of motion is restated in terms of momentum. The resulting relation is used, with Newton's third law, to introduce the general law of conservation of momentum for systems of any number of particles. An implication of the law of conservation of momentum is that the total momentum after a collision, if no external forces are applied to the system, is equal to the total momentum before the collision. This is useful because the equality holds independently of the details of the (possibly unknown or complicated) collision process.

The theory is applied to spacecraft propulsion.

Key Ideas

Students should know and understand the following:

Vector Form of Newton's Second Law of Motion

The acceleration $\vec{a}$ of an object is in the direction of the net force $\vec{F}$ acting on it.

Newton's second law of motion can therefore be expressed as a vector relation, $\vec{F} = m\vec{a}$.

Newton's Second Law of Motion in Terms of Momentum

Newton's second law of motion can be expressed as a vector relation, $\vec{F} = \Delta\vec{p}/\Delta t$, where $\vec{p} = m\vec{v}$ is the momentum of the object.

Intended Student Learning

Students should be able to do the following:

Vector Form of Newton's Second Law of Motion:

Given the initial velocity of a particle bouncing off a surface without a change of speed, and the duration of the collision, calculate the average acceleration of the particle.

Using $\vec{F} = m\vec{a}$, calculate the average force applied to the particle by the surface.

Using Newton's third law, deduce the average force applied to the surface by the particle.

Newton's Second Law of Motion in Terms of Momentum:

Derive $\vec{F} = \Delta\vec{p}/\Delta t$ by substituting the defining expression for acceleration ($\vec{a} = \Delta\vec{v}/\Delta t$) into Newton's second law of motion $\vec{F} = m\vec{a}$ for particles of fixed mass. (The net force $\vec{F}$, and hence the acceleration $\vec{a}$, are assumed to be constant. Otherwise, average or instantaneous quantities apply.)

Draw a vector diagram in which the initial momentum is subtracted from the final momentum, giving the change in momentum $\Delta\vec{p}$.

Key Ideas

Students should know and understand the following:

Law of Conservation of Momentum

Newton's third law of motion, $\vec{F}_1 = \vec{F}_2$, in conjunction with the second law expressed in terms of momentum, implies that the total momentum of a system of two interacting particles, subject only to the force of each one on the other, is conserved.

This can be extended to the law of conservation of momentum, which states that the total momentum of any number of objects remains unchanged in the absence of external forces.

Application: Spacecraft Propulsion

Intended Student Learning

Students should be able to do the following:

Solve problems involving the use of the vector relation $\vec{F} = \Delta\vec{p}/\Delta t$.

Derive an equation expressing the conservation of momentum for two interacting particles by substituting $\vec{F}_1 = \Delta\vec{p}_1/\Delta t$ and $\vec{F}_2 = \Delta\vec{p}_2/\Delta t$ into $\vec{F}_1 = -\vec{F}_2$.

Compare the magnitudes and directions of the total momentum vectors before and after a two-puck air-table collision recorded using a multi-image photograph, in order to show that momentum is conserved. Consider only examples in which the mass of one puck is an integral multiple of the mass of the other puck. Ignore the scale of the photograph, the flash rate, and the actual masses of the pucks.

Using trigonometric relations or scale diagrams, perform calculations in one or two dimensions, applying the law of conservation of momentum to two objects or to one object that explodes into two or three fragments.

Explain qualitatively, in terms of the law of conservation of momentum, the change in motion (in a straight line) of a spacecraft as a result of the emission of discrete particles (e.g. ions emitted by an ion thruster on a satellite).

Explain qualitatively, in terms of the law of conservation of momentum, how the reflection of light particles (photons) can be used to accelerate a solar sail.

Use vector diagrams to compare the acceleration of a spacecraft using a solar sail where photons are reflected with the acceleration of a spacecraft using a solar sail where photons are absorbed.

Section 2: Electricity and Magnetism

This section introduces the concept of fields as used in physics. The conventions adopted to represent fields pictorially show the magnitude and direction of the relevant field vectors at points within the field. Forces between stationary charges are discussed and the motion of charged particles in uniform electric fields is analysed quantitatively, in one and two dimensions. Comparisons are made with projectile motion, as described in Section 1.

Moving charges are also examined, first in electric currents and then in a vacuum. A magnetic field is shown to exist in each case. This magnetic field can exert a force on another electric current or a charge moving in a vacuum. In the latter case the force can cause the charge to move uniformly in a circle. The quantitative analysis of this motion involves the ideas on uniform circular motion developed in Section 1.

Topic 1: Electric Fields

The two fundamental postulates of electrostatics are introduced: Coulomb's law, and the principle of superposition. Several important electric field distributions are discussed.

The electric field at a point in space is defined and used, with Coulomb's law, to derive an expression for the electric field at a distance from a point charge. In this topic the charges are assumed to be in a vacuum (or, for practical purposes, air).

The principle of superposition is used to explain the fact that a near-uniform electric field can be produced by two charged parallel conducting plates. The absence of an electric field in hollow conductors is discussed. The presence of strong electric fields in the vicinity of sharp points on charged conductors is identified and applied to corona discharges in relation to photocopiers and laser printers.

Key Ideas	Intended Student Learning
Students should know and understand the following:	*Students should be able to do the following:*
Coulomb's Law	
Any two stationary point charges experience mutual forces along the line joining them. These forces are attractive if the charges are unlike and repulsive if they are alike. The magnitude of these forces is directly proportional to the product of the two charges and inversely proportional to the square of the distance between them.	Solve problems involving the use of $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$ where F is the magnitude of the electric forces, q_1 and q_2 are the charges, r is the distance between them, and $1/4\,\pi\varepsilon_0$ is the proportionality constant. Using proportionality, discuss changes in the magnitude of the force on each of the charges as a result of a change in one or both of the charges and/or a change in the distance between them. Explain that the electric forces are consistent with Newton's third law.

Key Ideas	Intended Student Learning
Students should know and understand the following:	*Students should be able to do the following:*
Principle of Superposition	
When more than two point charges are present, the force on any one of them is equal to the vector sum of the forces due to each of the other point charges.	Using vector addition, calculate the magnitude and direction of the force on a point charge due to two other point charges.
Electric Field	
Electric charges establish an electric field $\vec{E}$ in the surrounding space. The electric field at any point produces a force on an electric charge placed at that point. The electric field at a point is defined as the electric force $\vec{F}$ per unit charge on a small positive test charge q placed at that point, provided that all charges remain undisturbed: $\vec{E} = \vec{F}/q$. The direction of the electric force on a charge is parallel to the electric field if the charge is positive and antiparallel if the charge is negative.	Describe how the concept of the electric field replaces the concept of action at a distance (inherent in Coulomb's law) with the localised action of the field of one charge on the other charge. Solve problems involving the use of $\vec{E} = \vec{F}/q$. Determine the direction of the electric field at any point due to a point charge. Using Coulomb's law, derive the expression $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ for the magnitude of the electric field at a distance r from a point charge q. Solve problems involving the use of $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$.
Pictorial Representation of Electric Fields	
An electric field can be represented by field lines such that the direction of the field is at a tangent to each line, and the magnitude of the field at any point is represented by the number of lines crossing a unit area perpendicular to the field in the vicinity of the point.	Sketch the electric field lines for an isolated positive or negative point charge.
Superposition of Electric Fields	
The principle of superposition applies also to electric fields, as the electric field is the force per unit charge.	Calculate the magnitude and direction of the electric field at a point due to two charges with the same or opposite sign. Sketch the electric field lines for two point charges of equal magnitude with the same or opposite sign.

Key Ideas	Intended Student Learning
Students should know and understand the following:	*Students should be able to do the following:*

Electric Field Due to One or Two Charged Plates

The electric field due to an infinite charged conducting plate is uniform. The electric field between two infinite parallel conducting plates with equal and opposite charges per unit area is uniform between the plates and zero elsewhere. The electric field near and beyond the edges of two finite plates is non-uniform.	Describe and draw the electric field due to an infinite conducting plate of positive or negative charge. Using the principle of superposition, draw the electric field due to two infinite parallel conducting plates with equal and opposite charges per unit area. Sketch the electric field between and near the edges of two finite oppositely charged parallel plates.

Electric Fields and Conductors

For any conductor, whether charged or uncharged: • electric fields always meet the conducting surface at right angles • there is no electric field inside the conducting material. An uncharged conductor in an external electric field will experience charge polarisation.	In terms of the motion of the charges in the conductor, explain why: • the component of the electric field parallel to the conducting surface must be zero • there is no electric field inside the conducting material. Sketch the electric field that results when a solid uncharged conducting sphere is placed in the region between two oppositely charged parallel plates.

Electric Field Inside a Hollow Conductor

There is no electric field inside a hollow conductor of any shape, whether or not the conductor is charged, provided that there is no charge in the cavity.	Sketch the electric field produced by a hollow spherical charged conductor.

Electric Fields Near Sharp Points

Electric fields are strongest near sharp points on conductors. These fields may be large enough to ionise the air in the vicinity of the sharp points, resulting in charge movement away from the conductor. This is called a 'corona discharge'.	Sketch the electric field produced by a charged pear-shaped conductor. Describe how the large electric field in the vicinity of sharp points may ionise the air.

Key Ideas

Students should know and understand the following:

Application: Photocopiers and Laser Printers

Intended Student Learning

Students should be able to do the following:

Describe the action of a corona wire in charging the photoconductive surface of a photocopier or laser printer.

Describe the action of the corona wire in:

- charging the paper so as to transfer the toner from the photoconductive surface to the paper
- discharging the paper so that it does not cling to the photoconductive surface.