Projectile ***SOLUTIONS***

1.

a) $s_H = 10$ m $\quad v = ? \quad \theta = 70° \quad a_V = -g$

$v_{0_V} = v_0 \sin 70°$

Since the object lands at its launch height, $v_V = -v_{0_V}$

$v_V = v_{0_V} + a_V t$

$\therefore -v_{0_V} = v_{0_V} - gt$

$\therefore 2v_{0_V} = gt$

$\therefore t = \dfrac{2v_{0_V}}{g}$

$\therefore t = \dfrac{2v_0 \sin 70°}{g}$

b) $s_H = v_H t$

$\therefore v_H = \dfrac{s_H}{t}$

$\therefore v_H = \dfrac{s_H\, g}{2v_0 \sin 70°}$

c) Initial and final horizontal components of velocity are the same, so $v_H = v_{0_H} = v_0 \cos\theta$

From part b, $v_H = \dfrac{s_H\, g}{2v_0 \sin 70°}$

$\therefore v_0 \cos 70° = \dfrac{s_H\, g}{2v_0 \sin 70°}$

$\therefore {v_0}^2 = \dfrac{s_H\, g}{2\cos 70° \sin 70°}$

Speed can't be negative, so $v_0 = \sqrt{\dfrac{s_H\, g}{2\cos 70° \sin 70°}}$

$\therefore v_0 = \sqrt{\dfrac{10 \times 9.8}{2\cos 70° \sin 70°}} = 12.34 \text{ ms}^{-1}$

The boule's initial velocity will need to be 12 ms^{-1} at 70° above the horizontal.

2.

a) In order to find the horizontal range of the projectile, in this case we need the time of flight first.

$a_V = \text{-}9.8\text{ms}^{-2} \qquad s_V = \text{-}10\text{m} \qquad v_{0_V} = 0\text{ ms}^{-1} \qquad v_H = 7.0\text{ ms}^{-1} \qquad t = ?$

$$s_V = v_{0_V} t + \tfrac{1}{2} a_V t^2$$

$$\therefore s_V = \tfrac{1}{2} a_V t^2$$

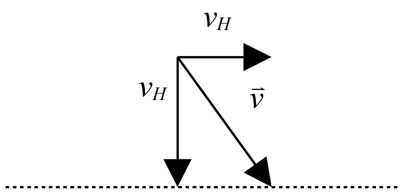
$$\therefore t = \sqrt{\frac{s_V}{\frac{1}{2} a_V}}$$

$$\therefore t = \sqrt{\frac{-10}{\frac{1}{2}\times(\text{-}9.8)}} = 1.4 \text{ s}$$

$$\begin{aligned} s_H &= v_H t \\ &= 7.0 \times 1.4 \\ &= 10 \text{ m} \end{aligned}$$

It is very likely the rock will hit the soldier.

b)



$$\begin{aligned} v_V &= v_{0_V} + a_V t \\ &= 0 + \text{-}9.8 \times 1.4 \\ &= \text{-}14 \text{ms}^{-1} \end{aligned} \qquad v_H = v_{0_H} = 7.0\text{ms}^{-1}$$

$$\begin{aligned} v &= \sqrt{(\text{-}14)^2 + 7.0^2} \\ &= 16 \text{ ms}^{-1} \end{aligned}$$

So the speed of the rock on impact is 16 ms^{-1}

3. $v_0 = 50\text{ms}^{-1} \quad \theta = 70° \quad v_V = 0\text{ms}^{-1} \quad s_V = ?$

$$v_{0_V} = v_0 \sin\theta = 50 \sin 70° = 47 \text{ ms}^{-1}$$

$$v_{0_H} = v_0 \cos\theta = 50 \cos 70° = 17 \text{ ms}^{-1}$$

$$v_V{}^2 = v_{0_V}{}^2 + 2 a_V s_V$$

$$\therefore s_V = \frac{v_V{}^2 - v_{0_V}{}^2}{2 a_V}$$

$$\therefore s_V = \frac{0^2 - 47^2}{2(-9.8)} = 113 \text{ m}$$

The maximum height the student reaches is 113 + 10 = 123 m

4.

$t = 5.62\,s \quad v_V = 0\,\text{ms}^{-1} \quad s_V = ?$

In half the time of flight, the Vortex would free fall the same distance as the maximum displacement here

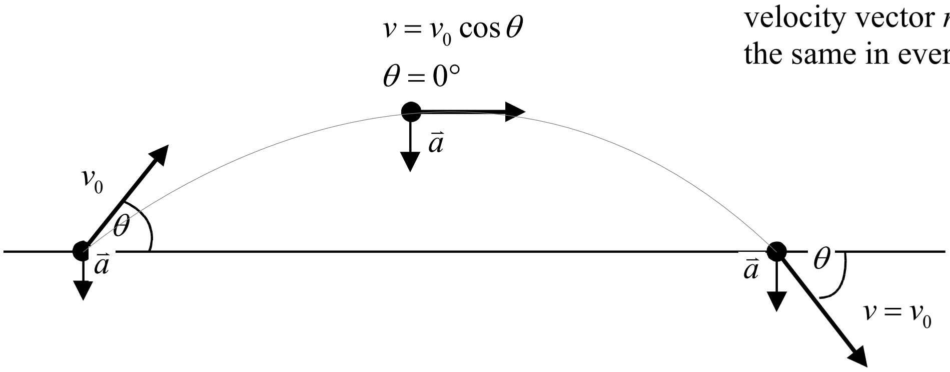
$$s_V = \tfrac{1}{2} a_V \left(\frac{t}{2}\right)^2$$

$$\therefore s_V = \tfrac{1}{2} \times (-9.8) \times \left(\frac{5.62}{2}\right)^2 = 38.69 \text{ m}$$

The maximum height is 1.59+38.69 = 40.3 m

5.

The horizontal size of the velocity vector *must* be the same in every case



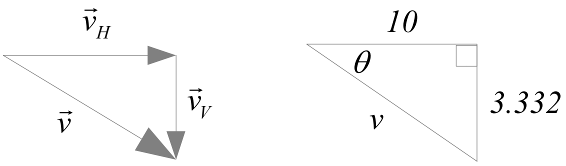
6.

(a) It is constant. The projectile moves an equal horizontal distance each equal time interval.

(b) $v_H = \dfrac{2.0}{0.20} = 10 \text{ ms}^{-1}$

$v_{0_V} = 0 \text{ ms}^{-1}$

$v_v = v_{0_V} + a_V t = 0 - 9.8 \times 0.34 = -3.332 \text{ ms}^{-1}$



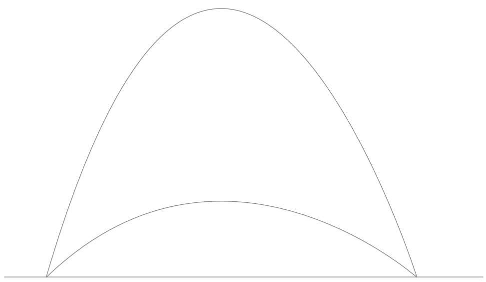
$$v = \sqrt{3.332^2 + 10^2} = 10.54 \text{ ms}^{-1}$$

$$\theta = \tan^{-1}\left(\frac{3.332}{10}\right) = 18.43°$$

The velocity of the projectile is 11 ms^{-1} at 18° below the horizontal.

7.

(a)



(b) One launch angle has more vertical component and less horizontal component, the other has more horizontal and less vertical. The angles are an equal magnitude away from the angle which would achieve maximum range (or they are complementary/add to 90 degrees).

8.

Air resistance provides a force on an object in the opposite direction to the object's motion and greater if the speed of the object is greater. Since the object's horizontal component will be slowed down, its range will be significantly reduced, according to $s_H = v_H t$. Since the object is launched horizontally, the time of flight will be increased because the vertical component of velocity is slowed.

9.

a)

$v_0 = 7.164\text{ms}^{-1} \quad \theta = 26.20° \quad a_V = \text{-}9.81\text{ms}^{-1}$

Distance between desks is range of projectile. To find range, we need time of flight.

$v_{0_V} = v_0 \sin\theta = 7.164 \sin 26.20° = 3.163\text{ms}^{-1}$

Find the time of flight when book is at level of desk ($s_V = 0$)

$s_V = v_{0_V} t + \tfrac{1}{2} a_V t^2$

$\therefore 0 = v_{0_V} t + \tfrac{1}{2} a_V t^2$

$\therefore t\left(v_{0_V} + \tfrac{1}{2} a_V t\right) = 0$

$\therefore v_{0_V} + \tfrac{1}{2} a_V t = 0 \text{ or } t = 0$

$$\therefore t = \frac{-v_{0_V}}{\tfrac{1}{2} a_V} = \frac{-3.163}{\tfrac{1}{2} \times (-9.8)} = 0.6455 \text{ s}$$

$s_H = v_H t$

$v_H = v_{0_H} = v_0 \cos\theta = 7.164 \cos 26.20° = 6.428\text{ms}^{-1}$

So $s_H = v_{0_H} t = 6.428 \times 0.6455 = 4.149 \text{ m}$

The desks are 4.149m apart

b)

Since the landing and launch heights are the same, the maximum height in this case occurs when t is half the time of flight, that is:

$t = \frac{1}{2} \times 0.6455 = 0.3227$

$$
\begin{aligned}
s_V &= v_{0_V} t + \tfrac{1}{2} a_V t^2 \\
&= 3.163 \times 0.3227 + \tfrac{1}{2}(\text{-}9.81) \times 0.3227^2 \\
&= 0.5104 \text{ m}
\end{aligned}
$$

The maximum height of the book is 0.5104 m above the desks (1.410 m above the ground)

10.

a) It has a rougher surface texture, and a greater projected area

b) It has less inertia (less mass)