Gravitation ***SOLUTIONS***

1.

a) $m_1 = 3.5\times10^5\,\text{kg} \quad m_2 = 8.9\times10^{23}\,\text{kg} \quad r = 4.1\times10^5\,\text{km} = 4.1\times10^8\,\text{m}$

$$\begin{aligned} F &= G\frac{m_1 m_2}{r^2} \\ &= 6.67\times10^{-11}\,\frac{3.5\times10^5\times8.9\times10^{23}}{\left(4.1\times10^8\right)^2} \\ &= 1.2\times10^2\,\text{N} \end{aligned}$$

Serenity experiences $1.2\times10^2\,\text{N}$ (2 s.f.) towards Rigel II.

b) 1.2×10^2N (2 s.f.) towards Serenity.

c) $m_1 = 3.5\times10^5\,\text{kg} \quad F = 1.2\times10^2\,\text{N}$

$$\begin{aligned} F &= m_1 a \\ \therefore a &= \frac{F}{m_1} \\ &= \frac{1.2\times10^2\,\text{N}}{3.5\times10^5\,\text{kg}} \\ &= 3.5\times10^{-4}\,\text{ms}^{-2} \end{aligned}$$

Serenity accelerates at $3.5\times10^{-4}\,\text{ms}^{-2}$ (2 s.f.) towards Rigel II.

d) There will be no effect. Gravitational acceleration only depends on the *other* body's mass, according to $g = G\dfrac{M}{r^2}$

2. *Some* possible options:

Geostationary	**Polar**
High orbit	Low orbit
Long period	Short period
West to east direction (same as Earth)	Close to north-south or south-north direction
Slower speed	Higher speed

3.

a) Equating $v = \sqrt{\frac{GM}{r}}$ and $v = \frac{2\pi r}{T}$ gives:

$$\frac{2\pi r}{T} = \sqrt{\frac{GM}{r}}$$

$$\therefore \left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$$

$$\therefore \frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

$$\therefore 4\pi^2 r^3 = T^2 GM \quad \{\text{multiplying both sides by } T^2 r\}$$

$$\therefore r^3 = \frac{T^2 GM}{4\pi^2}$$

$$\therefore r = \sqrt[3]{\frac{T^2 GM}{4\pi^2}}$$

b) $M = 5.97\times10^{24}$kg $R = 6.4\times10^{6}$m T = 24 hours = 8.64×10^{4}s $r = ?$

$$r = \sqrt[3]{\frac{T^2 GM}{4\pi^2}}$$

$$= \sqrt[3]{\frac{\left(8.64\times10^4\right)^2 \times 6.67\times10^{-11} \times 5.98\times10^{24}}{4\pi^2}}$$

$$= 4.22\times10^7 \text{ m (3 s.f.)}$$

So a geostationary satellite is at 4.22×10^7 m from the centre of the Earth.

The altitude is $4.22\times10^7 - 6.4\times10^6 = 3.6\times10^7$ m (2 s.f.) from the Earth's surface.

4.

$$F = G\frac{m_1 m_2}{r^2}$$

G, m_1 and m_2 are constant.

$$\therefore F \propto \frac{1}{r^2}$$

$\therefore F$ and $\frac{1}{r^2}$ are in constant ratio

$r_A = r,\ \ r_B = 3r$

$$\therefore \frac{F_A}{F_B} = \frac{1/r_A^{\,2}}{1/r_B^{\,2}} = \frac{1}{r_A^{\,2}} \times \frac{r_B^{\,2}}{1} = \frac{r_B^{\,2}}{r_A^{\,2}}$$

$$\therefore \frac{F_A}{F_B} = \frac{(3r)^2}{r^2} = \frac{9r^2}{r^2} = \frac{9}{1}$$

$$\therefore F_B = \frac{F_A}{9} = \frac{195}{9} = 21.7 \text{ N}$$

Optional revision solutions

1.

a) (one of various possible methods)
$t = 18.7\text{s}$ $\quad s_H = 1.98\times10^3\text{m}$ $\quad v_0 = 140\text{ms}^{-1}$ $\quad \theta = ?$

Follow the usual process for finding the range from time of flight, but use the pronumeral θ for the angle. Then rearrange to find θ.

$$v_H = v_{0_H} = 140\cos\theta$$

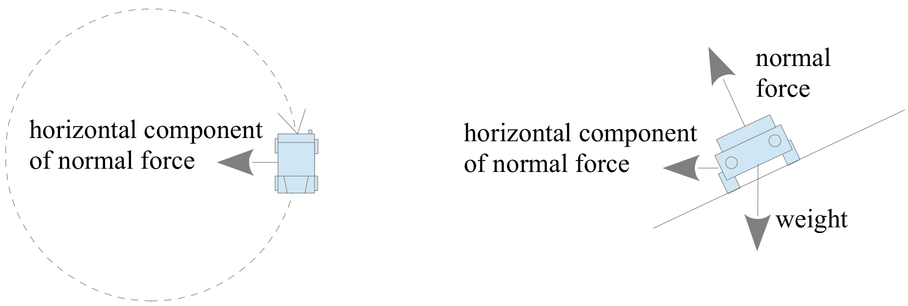
$$s_H = v_H t$$
$$\therefore\ 1.98\times10^3 = 140\cos\theta\times18.7$$
$$\therefore\ \cos\theta = \frac{1.98\times10^3}{140\times18.7}$$
$$\therefore\ \theta = \cos^{-1}\left(\frac{1.98\times10^3}{140\times18.7}\right)$$
$$= 40.8^\circ \text{ (3 s.f.)}$$

b) 49° (assuming the projectile is fired on level ground) – because the max range will be at 45° so 4° either side of this will be the same range (some range less than max).

c) The time of flight would be longer. The angle being higher means the vertical component of initial velocity would be larger, which increases time of flight.

2.

a)



(vectors not to scale)

The car is relying only on the normal force to accelerate, and the normal force is unchanged by the icy surface of the road (this only affects friction). Therefore, the car will continue to turn the corner unaffected.

b) (i) If the road was banked less steeply, the car would be relying on friction to provide some of the centripetal force. If the road became icy, the force of friction would be less, so the force necessary to accelerate into the corner would not be provided; the radius of the car's turn would increase (the car would go off the outside of the corner).

(ii) If the road was banked more steeply, the car would be relying on friction to stop it sliding down the banking. If the road became icy, this friction would no longer be provided, and the car would slide towards the inside of the corner.