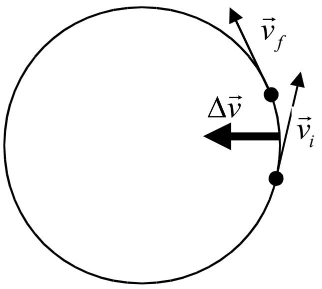
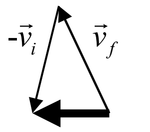
Uniform Circular Motion ***SOLUTIONS***

1. Take an object revolving in uniform circular motion. At one point it has velocity Δt later it has velocity $\vec{v}_f$. It has changed by $\Delta\vec{v}$.





$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$

(shown on circle diagram also)

The average acceleration $\vec{a}_{\text{average}} = \dfrac{\Delta\vec{v}}{\Delta t}$ which is in the same direction as the change in velocity.

$\left(\text{The formula could be rearranged to } \Delta\vec{v} = \vec{a}_{\text{average}}\Delta t\right)$

The average acceleration is perpendicular to the velocity exactly between the two velocities given. If the timescale were infinitesimal, the two velocities would occur at almost the same position. There would effectively be one velocity with acceleration exactly perpendicular to it; towards the centre. /4

2.

a)

$r = 22\text{m} \quad v = 15\text{ms}^{-1} \quad m = 1.2\times10^3\text{kg}$

$$F = \frac{mv^2}{r}$$
$$= \frac{1.2\times10^3\times15^2}{22}$$
$$= 1.23\times10^4\text{N}$$

So 1.2×10^4N of friction (2 s.f.) is required to keep his car on the road.

/2

b)

The rain reduces the amount of friction between the road and the tyres. The higher his speed, the more force is required to accelerate his car into the curve, according to $F = \dfrac{mv^2}{r}$. If he drives too fast, the wet road will not be able to provide F and he won't be able to turn the corner (he will crash).

/3

3.

a) The tension in the string. /1

b) The centripetal acceleration (ma_c) is provided only by the horizontal component of the string's tension, while the vertical component counteracts the weight (the force of gravity) of the ball. The total string tension (horizontal component plus vertical component) will be greater than the horizontal component by itself. /2

c) The vertical component of the string counteracts the weight (force of gravity).

$F = mg = 2.00\times9.8 = 19.6\text{N}$ (3 s.f.)

/2

4.

a) The period T is the time the object takes to traverse the circumference once. Therefore, since speed = distance ÷ time, we have that v = circumference ÷ T.

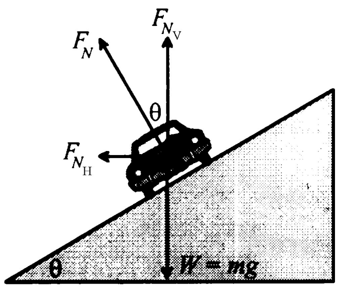
Circumference is $2\pi r$, so speed, radius and period are related by the equation $v = \dfrac{2\pi r}{T}$ /2

b) $r = 200\text{m} \quad T = 22.6\text{s} \quad v = ?$ /2

$$v = \frac{2\pi r}{T}$$
$$= \frac{2\pi \times 200}{22.6}$$
$$= 55.6\text{ms}^{-1}$$

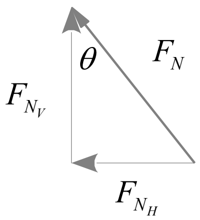
The Porsche is travelling at 55.6ms^{-1} (3 s.f.)

c)



The vertical component must still be sufficient to keep the car from sinking into the road, so $F_{N_V} = mg$.
For the horizontal component to provide exactly all the centripetal acceleration (friction of the tyres not needed) $F_{N_H} = F_c = ma_c = m\dfrac{v^2}{r}$

The total normal force is the vector sum of its components, so:



$$\tan\theta = \frac{F_{N_H}}{F_{N_V}}$$
$$\therefore \tan\theta = \frac{m\frac{v^2}{r}}{mg}$$
$$\therefore \tan\theta = \frac{v^2}{rg}$$

/4

d) $v = 55.6\text{ms}^{-1} \quad g = 9.8\text{ms}^{-2} \quad r = 200\text{m}$

$$\tan\theta = \frac{v^2}{rg}$$
$$= \frac{55.6^2}{200 \times 9.8}$$
$$= 57.6°$$

For no centripetal acceleration to be provided by friction, the curve must be banked at 57.6° (3 s.f.)

/2